一般的波动理论

波函数

基本概念

一种(广义)的物理场的时空分布

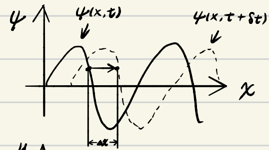
具体例子

- 简谐波(1D)

$$\psi(x,t) = \vec{A}e^{i(kx-\omega t)} = \vec{A}e^{i\varphi}$$

相速度 $= \frac{\text{位移}}{\text{时间}}$ $\quad \vec{v} = \frac{\omega}{|k|}\hat{k}$

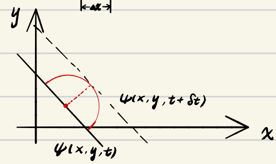
$$v = \left(\frac{\partial x}{\partial t}\right)_\varphi = \frac{-(\partial\psi/\partial t)_x}{(\partial\psi/\partial x)_t} = \frac{\omega}{k}$$



- 简谐波(2D)

$$\psi(x,y,t) = \vec{A}e^{i(k_x x+k_y y-\omega t)}$$

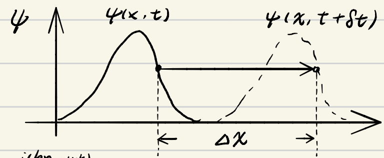
$$v = \frac{\omega}{\sqrt{k_x^2+k_y^2}}$$



- 脉冲波(1D)

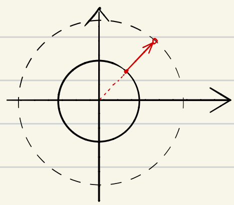
$$\psi(x,t) = \vec{A}\,\frac{1}{(x-ut)^2+1}$$

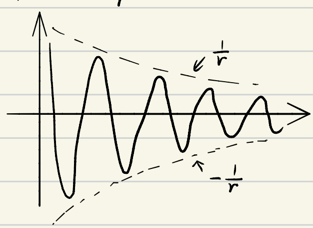
$$v = u$$



- 球面波(3D)

$$\psi(x,y,z,t) = \psi(r,\theta,\phi,t) = \frac{\vec{A}}{r}e^{i(kr-\omega t)}$$





能量守恒

$$\rho_E V = \rho_E \cdot 4\pi r^2 \delta r = const$$

$$\rho_E \propto |\psi(x,t)|^2$$

$$\frac{|\psi(x,t+\Delta t)|^2}{|\psi(x,t)|^2} = \frac{R^2(t)}{R^2(t+\Delta t)}$$

$$|\psi(x,t)| \propto \frac{1}{R}$$

$$\begin{cases} \nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \\ \nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) = \frac{\partial^2\psi}{\partial r^2} + \frac{2}{r}\frac{\partial\psi}{\partial r} = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) \end{cases}$$

$$\Rightarrow \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \Rightarrow \frac{\partial^2}{\partial r^2}(r\psi) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(r\psi)$$

$$\Rightarrow r\psi(\vec{r},t) = Ae^{i(\vec{k}\cdot\vec{r}-\omega t)} \quad \Rightarrow \psi(\vec{r},t) = \frac{A}{r}e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

波动方程

推导

① 寻找对偶场　$\vec{A}$ v.s. $\vec{B}$

② 建立耦合关系　$\partial_t \vec{A} = \alpha \partial_x \vec{B}$　　$\partial_t \vec{B} = \beta \partial_x \vec{A}$　　时间偏导、空间偏导

③ 解耦合　$\partial_t^2 \vec{A} = \alpha \partial_x \partial_t \vec{B} = \alpha\beta \partial_x^2 \vec{A}$

④ 波源　$\underbrace{\partial_t^2 \psi(x,t)}_{\text{加速度}} = v^2 \partial_x^2 \psi(x,t) + \underbrace{f(t)\delta(x-x_0)}_{S(x,t)}$ 驱动力

求解 (以1D简谐波为例)

$\partial_t^2 \psi(x,t) = v^2 \partial_x^2 \psi(x,t)$

通解　$\psi(x,t) = f(x+vt) + g(x-vt)$

特解　$\psi(x,t) = \vec{A} e^{i(kx-\omega t)}$

参数制约　波动方程的形式决定了波函数的形式

波动方程的参数决定了波函数的参数

特例

1D简谐波

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \cancel{\frac{\partial^2}{\partial y^2}} + \cancel{\frac{\partial^2}{\partial z^2}}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \Rightarrow \psi(x,t) = \vec{A} e^{i(kx-\omega t)}$$

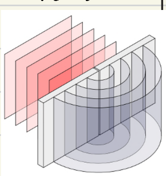
球面

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \cancel{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)} + \cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}}$$

$$\frac{\partial^2}{\partial r^2}(r\psi) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(r\psi) \Rightarrow \psi(r,t) = \frac{\vec{A}}{r} e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

柱面

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}} + \cancel{\frac{\partial^2}{\partial z^2}}$$



$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$　　$\psi(r,t) = \varphi(r)T(t)$

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{d\varphi(r)}{dr}\right] - \lambda\varphi(r) = 0$$　　Bessel 方程

$$\psi(r,t) \approx \frac{\vec{A}}{\sqrt{r}} e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

电磁波

波函数

光学只研究平面波
$$\begin{cases} \vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \\ \vec{B}(\vec{r},t) = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \end{cases}$$

参数制约关系

波函数参数 $(\mu, \varepsilon) \Rightarrow (\vec{E}_0, \vec{B}_0, \vec{k}, \omega) = (|E_0|, \hat{E}_0, |B_0|, \hat{B}_0, |k|, \hat{k}, \omega)$

	M1	M2	M3	M4
$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$	$\hat{E}_0 \cdot \hat{k} = 0$ $\vec{\nabla}\cdot\vec{E}_0 = 0$		方向：$\hat{k}\times\hat{E}_0 = -\hat{B}_0$ 大小：$\lvert k\rvert\cdot\lvert E_0\rvert = \omega\lvert B_0\rvert$ $\lvert k\rvert\cdot\lvert B_0\rvert = \mu\varepsilon\omega\lvert E_0\rvert$	$\hat{k}\times\hat{B}_0 = \hat{E}_0$ $\Rightarrow \frac{\omega}{\lvert k\rvert} = \frac{1}{\sqrt{\mu\varepsilon}} = c$ $\frac{\lvert E_0\rvert}{\lvert B_0\rvert} = \frac{1}{\sqrt{\mu\varepsilon}} = c$
$\vec{B}(\vec{r},t) = \vec{B}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$		$\hat{B}_0 \cdot \hat{k} = 0$ $\vec{\nabla}\cdot\vec{B} = 0$	$\vec{\nabla}\times\vec{E} = -\partial_t \vec{B} \Rightarrow$ 方向、大小相同	

光的物理量

能量密度 w

单位体积的辐射能量

$w = \frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2$ (对偶场模方和) $= \varepsilon E^2 = \frac{1}{\mu}B^2$

能流密度 (Poynting 矢量) $\vec{S}$ $[\frac{W}{m^2}]$

单位时间通过单位面积传输的能量

$\vec{S} = w\vec{v}$

$\vec{\nabla}\cdot\vec{S} = -\partial_t w$

= 看在光的情形下等价

$S = \frac{w v \Delta t A}{\Delta t A} = wv$

$\vec{S} = \vec{E}\times\vec{H}$

动量密度 $\vec{p}$

质能关系 $E = mc^2 \Rightarrow w = \rho c^2$

电磁场有质量 $\vec{p} = \rho\vec{v} = \frac{w}{c^2}\vec{v} = \frac{\vec{S}}{c^2} = \frac{1}{c^2}\vec{E}\times\vec{H}$

辐照度 $I = \langle S\rangle_T$

Poynting 矢量的时间平均值

$I = \langle S\rangle_T = \frac{c\varepsilon_0}{2}E_0^2$

Maxwell's Equation

$$\begin{cases} \text{M1} & \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon} \\ \text{M2} & \vec{\nabla}\cdot\vec{B} = 0 \\ \text{M3} & \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \\ \text{M4} & \vec{\nabla}\times\vec{B} = \mu(\vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t}) \end{cases}$$

$\Rightarrow$

Electric - Magnetic Equation

$$\begin{cases} \partial_t^2\vec{E} = \frac{1}{\varepsilon\mu}\nabla^2\vec{E} \\ \partial_t^2\vec{B} = \frac{1}{\varepsilon\mu}\nabla^2\vec{B} \end{cases}$$

波动方程

对偶场

电场 v.s. 磁场

耦合关系

Maxwell's equation $(\rho = 0,\ \vec{J} = 0)$

$$\vec{\nabla}\times\vec{E} = -\partial_t\vec{B} \qquad \vec{\nabla}\times\vec{B} = \mu\varepsilon\,\partial_t\vec{E}$$

$$\vec{\nabla}\cdot\vec{E} = 0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$

解耦合

$$\begin{cases} \vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\nabla^2\vec{E} = -\partial_t(\vec{\nabla}\times\vec{B}) = -\mu\varepsilon\,\partial_t^2\vec{E} \\ \vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B} = -\nabla^2\vec{B} = \mu\varepsilon\,\partial_t(\nabla\times\vec{E}) = -\mu\varepsilon\,\partial_t^2\vec{B} \end{cases}$$

$$\Rightarrow \quad \nabla^2\vec{E} = \mu\varepsilon\,\partial_t^2\vec{E} \qquad \nabla^2\vec{B} = \mu\varepsilon\,\partial_t^2\vec{B}$$

$$\nabla^2\vec{E} = \frac{1}{v^2}\partial_t^2\vec{E} \quad \Rightarrow \quad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

波源

辐射

光的性质

辐射：光是怎么产生的

辐射

加速运动的电荷辐射电磁波

波动方程的波源项 $\quad \partial_t^2 \vec{E} = c^2 \nabla^2 \vec{E} + \text{Source}$

数学描述

波动方程

Maxwell's Equation $\begin{cases} \vec{\nabla} \cdot \vec{E} = \dfrac{\rho}{\varepsilon} & \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{E} = -\partial_t \vec{B} & \vec{\nabla} \times \vec{B} = \mu(\vec{J} + \varepsilon \partial_t \vec{E}) \end{cases}$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = \frac{1}{\varepsilon} \nabla \rho - \nabla^2 \vec{E}$$

$$= -\partial_t (\vec{\nabla} \times \vec{B}) = -\mu \partial_t \vec{J} - \mu\varepsilon \partial_t^2 \vec{E}$$

$$\Rightarrow \boxed{\partial_t^2 \vec{E} = c^2 \nabla^2 \vec{E} - \frac{1}{\varepsilon}(\partial_t \vec{J} + c^2 \nabla \rho)}$$

电流密度 $\vec{J} = ne\vec{v}$ $\quad \vec{J} = nq\underset{\text{加速度}}{\underline{\dot{\vec{v}}}} + 0$

系统

① 加速运动的点电荷

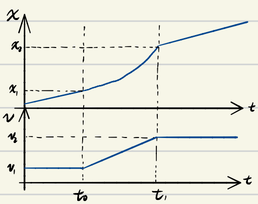
$$\begin{cases} \rho(\vec{r}, t) = \delta(\vec{r} - \vec{r}(t)) \\ \vec{J}(\vec{r}, t) = \rho q \vec{v} = q\vec{v}(t)\delta(\vec{r} - r(t)) \end{cases}$$

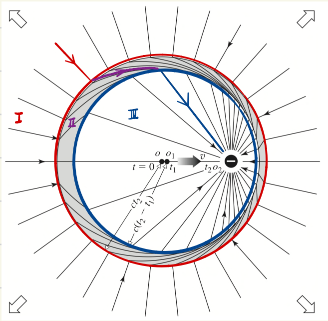
② 导线中的加速电流

$$\begin{cases} \rho(\vec{r}, t) = 0 \\ \vec{J}(\vec{r}, t) = \underbrace{\rho_+ q \vec{v}_+}_{=0} + \rho_- q \vec{v}_- = \rho_- q \vec{v}_- \end{cases}$$

例子（矢性偏振、辐照度）

匀加速运动点电荷





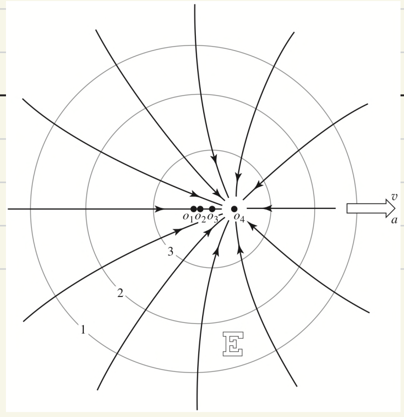
① 电荷对电场的扰动以光速传播

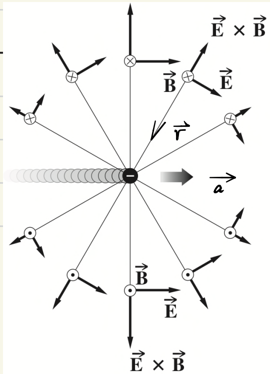
○ 以外的 I 区感受到的是 t_0 时刻前的电场

○ 以内的 III 区感受到的是 t_1 时刻后的电场

② 电场线是连续的

连接 I 区和 III 区对应的电场线得到 II 区





$$\vec{B} \propto \vec{a} \times \vec{r}$$

$$\vec{E} \propto \vec{r} \times (\vec{r} \times \vec{a})$$

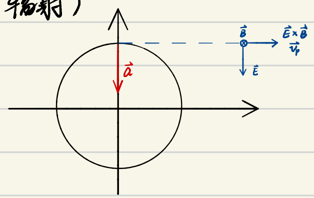
辐照度

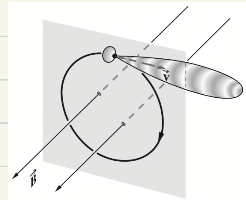
$I(0) = I(180^\circ) = 0$

$I(90^\circ) = I(270^\circ) = I_{max}$

圆周加速运动

（同步辐射）

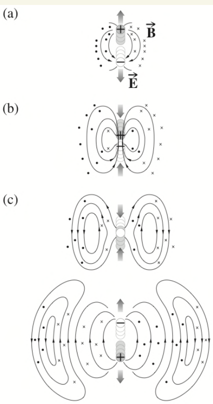




θ：观察点与加速度方向的夹角

$I(\theta)$： $I(0)=I(180^\circ)=0$ $I(90^\circ)=I(180^\circ)=I_{max}$

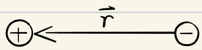
偶极辐射



偶极子

偶极振荡

偶极辐射

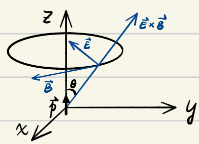


偶极距 $\vec{p}=q\vec{r}$

① 电场线总是从正电荷指向负电荷 或首尾相接成闭环

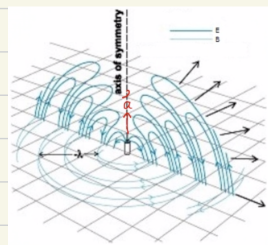
② 电场线不会无缘无故地消失（电场携带能量）

③ 电荷对电场的扰动以光速传播



$\vec{B}\propto\vec{a}\times\vec{r}$

$\vec{E}\propto\vec{r}\times(\vec{r}\times\vec{a})$



远场近似（$kr>>1 \Rightarrow r>>\frac{1}{k}$）

$$E=\frac{p_0k^2\sin\theta}{4\pi\varepsilon_0 r}e^{i(kr-\omega t)} \quad \propto \sin\theta$$

$$I(\theta)=\frac{1}{T}\int_0^T S\,dt=\frac{p_0^2\omega^4}{32\pi^2c^3\varepsilon_0}\frac{\sin^2\theta}{r^2} \quad \propto \sin^2\theta$$

设偶极子 $\vec{p}=\vec{p}_0e^{i(kr-\omega t)}$ $\vec{p}_0=p_0\cos\theta\,\hat{r}-p_0\sin\theta\,\hat{\theta}$

电场推迟势 $\vec{A}=-\frac{i\mu\omega p_0e^{i(kr-\omega t)}}{4\pi r}(\cos\theta\,\hat{r}-\sin\theta\,\hat{\theta})$

$$\vec{B}=\vec{\nabla}\times\vec{A}=\frac{1}{r^2\sin\theta}\begin{vmatrix}\hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\varphi}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi}\\ A_r & A_\theta & A_\varphi\end{vmatrix}=\frac{\mu k^2\omega p_0}{4\pi}\left(\frac{1}{kr}-\frac{i}{k^2r^2}\right)\sin\theta\, e^{i(kr-\omega t)}\,\hat{\varphi}$$

$$\vec{\nabla}\times\vec{H}=\frac{\partial\vec{D}}{\partial t} \Rightarrow \nabla\times\vec{B}=\mu\varepsilon\frac{\partial\vec{E}}{\partial t}=-i\omega\mu\varepsilon\vec{E}$$

$$\vec{E}=-\frac{1}{i\omega\mu\varepsilon}\vec{\nabla}\times\vec{B}=\frac{k^3p_0}{4\pi\varepsilon}\left\{\left[\frac{2i}{(kr)^2}+\frac{i}{(kr)^3}\right]\cos\theta\,\hat{r}+\left[-\frac{1}{kr}+\frac{i}{(kr)^2}+\frac{1}{(kr)^3}\right]\sin\theta\,\hat{\theta}\right\}e^{i(kr-\omega t)}$$

色散

概念

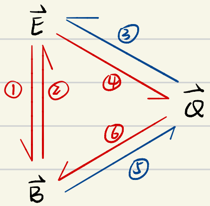
不同颜色的光在介质中发生分散

不同频率的光在介质中折射率不同

介面

波动方程

耦合场



① M4 $\nabla\times\vec{B} = \mu(\vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t})$

② M3 $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

③ M1 $\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon}$

④ 库仑定律 $\vec{E} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}\hat{r}$

⑤ 洛伦兹力 $\vec{f} = \rho\vec{E} + \vec{J}\times\vec{B}$

⑥ M4 $\nabla\times\vec{B} = \mu(\vec{J} + \varepsilon\frac{\partial\vec{E}}{\partial t})$

波动方程

微观世界

(单个电偶极子) 电偶极振荡函数

① 研究对象 x

② 建立方程(驱动) $m_e\partial_t^2 x = -m_e\omega_0^2 x + qE_0\cos\omega t$

③ 求解方程

通解 $x(t) = \alpha\cos\omega_0 t + \alpha'\sin\omega_0 t + \beta\cos\omega t + \beta'\sin\omega t$

特解 $\begin{cases} \text{初始条件 } x(t=0) \text{ 的位移和速度在物理上不重要 } \alpha = \alpha' = 0 \\ \text{驱动同相位 } \beta' = 0 \end{cases}$

$\Rightarrow x(t) = \frac{q/m_e}{\omega_0^2-\omega^2}E_0\cos\omega t$

$\vec{P} = Nq\vec{r}$

宏观世界

电介质

① 色散来自于折射率 $n = n(\omega)$

② $n = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} \approx \sqrt{\frac{\varepsilon}{\varepsilon_0}}$

③ 介电材料中电偶极子贡献 $E_{dipole} \Rightarrow \varepsilon$

$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon\vec{E} \Rightarrow \varepsilon = \varepsilon_0 + \frac{P}{E}$

波函数

物理性质

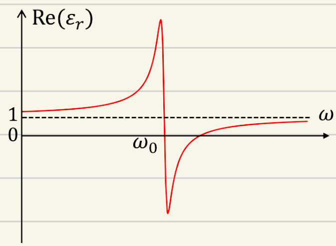
色散

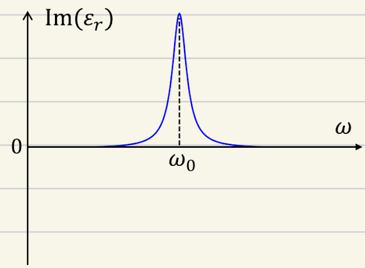
从实验/经验可知 $\psi = E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$$m_e \frac{d^2x}{dt^2} = f + F + F_{dis} = -m_e\omega_0^2 x + q_e E_0\cos\omega t - m_e\gamma\frac{dx}{dt}$$

$$n^2(\omega) = 1 + \frac{Nq_e^2}{\varepsilon_0 m_e}\left(\frac{1}{\omega_0^2-\omega^2+i\gamma\omega}\right) = 1+\frac{\beta}{\omega_0^2-\omega^2+i\gamma\omega}$$

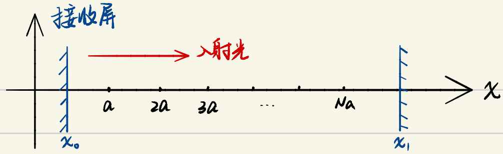
$$\mathrm{Re}(n^2) = 1 + \frac{\beta(\omega_0^2-\omega^2)}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2} \qquad \mathrm{Im}(n^2) = \frac{-\beta\gamma\omega}{(\omega_0^2-\omega^2)^2-\gamma^2\omega^2}$$





光在介质中的传播：散射 & 干涉

基本物理过程	散射与干涉的竞争与合作
合作	先单偶极子散射再集体干涉
竞争	谁起主导作用 { 介质密度小时，散射主导 介质密度大时，干涉主导　　判据①
散射	单个偶极子的"受激辐射"
过程	偶极子激发 + 偶极子辐射　($\omega_i = \omega_{偶极} = \omega_0$)
散射的频率依据	判据②
	偶极激发　$\vec{u} = \frac{\beta \vec{E}}{\omega_0^2 - \omega^2}$
	入射光 ω 越靠近 ω_0，激发越大
	偶极辐射　$I = \frac{p_0^2 \omega^4}{32\pi^2 c^3 \varepsilon_0} \frac{\sin^2\theta}{r^2}$
	入射光 ω 越大，辐射越强
例子	瑞利散射
	稀薄外层大气 $\Rightarrow$ 判据① $\Rightarrow$ 散射主导
	$\omega_{可见光} < \omega_{空气频率}$ $\Rightarrow$ 判据② $\Rightarrow$ 蓝光更散射
	散射体尺度小 $d \sim \frac{\lambda}{15}$
干涉	
过程	多个偶极子的辐射波之间的干涉
判据③	入射光方向决定相长/相消干涉方向
	相长干涉　顺着入射光向 } 光在介质中总是向前传播
	相消干涉　逆着，侧向 } (尽管有时向后散射，但向后的散射干涉相消)
数学证明	一维系统中的光向前相长，向后相消
	(见下页*)



*干涉的相长相消方向

系统

入射光波函数 $\vec{E} = \vec{E}_0 e^{i(kx-wt)}$

散射体(偶极子)位置 $x_n = na$

动力学过程

光传播

入射光激发偶极振荡① ⇒ 偶极辐射出次级波② ⇒ 各次级波在接收屏上干涉③

①偶极振荡

u_n：第 n 个偶极子偏离平衡位置的位移 $\vec{p} = q\vec{u}_n$

动力学方程：$m\ddot{u}_n = -mw_0^2 u_n + \underbrace{\vec{E}_0 e^{i(kna-wt)}}_{\vec{E}(na,t)}$

u_n的解：$u_n(t) = \dfrac{\beta e^{i(kna-wt)}}{w_0^2-w^2}$

②偶极辐射

$\vec{E}_n(x,t)$ 第 n 个偶极子发射的电磁波

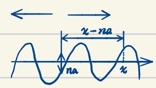
波动方程 $\partial_t^2 \vec{E}_n \approx v^2\partial_x^2 \vec{E}_n + \vec{j} = v^2\partial_x^2 \vec{E}_n + q\ddot{\vec{u}}_n \delta(x-na)$

$\vec{E}_n(x,t)$的解 $\vec{E}_n(x,t) \propto \vec{E}_n^+(x,t) + \vec{E}_n^-(x,t)$ （通解 $f(x+vt)+g(x-vt)$）

$\vec{E}_n^{\pm}(x,t) \propto e^{i[\underbrace{\pm k(x-na)}_{①}-wt+\underbrace{kna}_{②}+\underbrace{\pi}_{③}]}$

① $\pm$ ~ 左/右行波

$(x-na)$ ~ 从波源$(x=na)$发出的波的相位积累



② 继承自源点处偶极振荡 u_n 的自带相位

③ $\ddot{\vec{u}}_n$ 相对于 $\vec{u}_n$ 的相位延迟 $\ddot{\vec{u}}_n \propto -e^{i(kna-wt)} = e^{i(kna-wt+\pi)}$

③干涉

前屏上的干涉$(x=x_1)$：右行波的相加

$E_{前}(x_1,t) = \sum_n E_n^+(x_1,t) = \sum_n E_0 e^{i[k(x_1-na)-wt+kna+\pi]} = E_0 \sum_n e^{i(kx_1-wt+\pi)} = nE_0 e^{i(kx_1-wt+\pi)}$

⇒ n个子波均同相，相长干涉

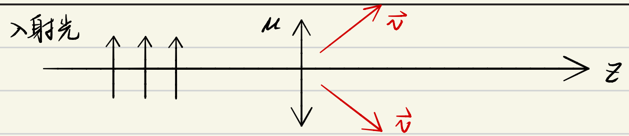
后屏上的干涉$(x=x_0)$：左行波的相加

$E_{后}(x_2,t) = \sum_n E_n^-(x_2,t) = \sum_n E_0 e^{i[-k(x_2-na)-wt+kna+\pi]} = E_0 e^{i(-kx_2-wt+\pi)} \sum_n e^{i2kna}$

⇒ n个子波，相邻者相位相差 $2kna$

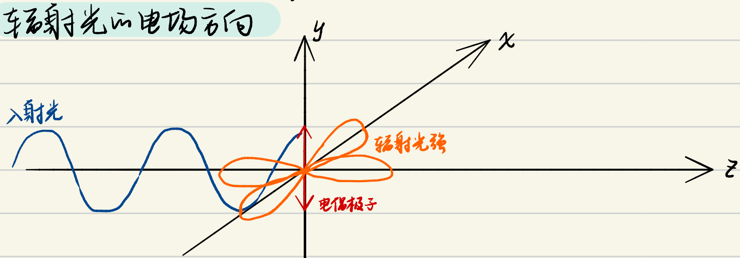
散射起偏

散射的基本物理过程

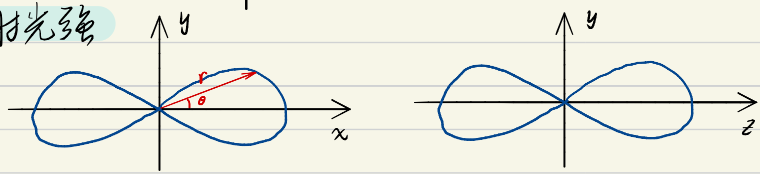


- 过程① 入射光激发散射子(电偶子)的振荡 ⇒ 偶极激发
- 过程② 电偶子激发后会向四周发射次级波 ⇒ 偶极辐射

辐射光的电场方向

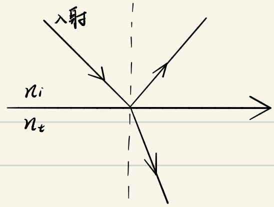


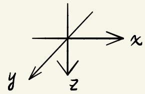
辐射光强



$\begin{cases} r: \text{光的强度} \\ \theta: \text{光的传播方向} \end{cases}$

散射光传播方向 / 入射光偏振方向	x 方向	y 方向	z 方向
y-偏振	有散射,散射光偏振方向为y方向	无散射	有散射 y偏振
x-偏振	无散射	有散射 x偏振	有散射 x偏振
自然光 (有x、y-偏振)	有散射 y偏振	有散射 x偏振	有散射 偏振有x、y方向
	y-线偏光	x-线偏光	自然光





光的传播

折射和反射

入射面为 x, z 平面 ($k_i^y = 0$)

物理系统

已知：入射光的波函数

$$\vec{E}_i = \vec{E}_i^0 e^{i(\vec{k}_i\cdot\vec{r} - w_i t)} = (E_i^{0x}, E_i^{0y}, E_i^{0z}) e^{i(k_i^x x + k_i^z z - w_i t)}$$

求：{ 反射光的波函数，折射光的波函数

$$\vec{E}_r = \vec{E}_r^0 e^{i(\vec{k}_r\cdot\vec{r} - w_r t)} = (E_r^{0x}, E_r^{0y}, E_r^{0z}) e^{i(k_r^x x + k_r^z z - w_r t)}$$

$$\vec{E}_t = \vec{E}_t^0 e^{i(\vec{k}_t\cdot\vec{r} - w_t t)} = (E_t^{0x}, E_t^{0y}, E_t^{0z}) e^{i(k_t^x x + k_t^z z - w_t t)}$$

参数个数 {

入射光：$(\vec{E}_i^0, \vec{k}_i, w_i) = (E_i^{0x}, E_i^{0y}, E_i^{0z}, k_i^x, k_i^y = 0, k_i^z, w_i)$

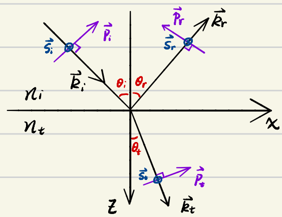
总参数 6　　自由参数 4

反射/折射光：$(\vec{E}_r^0, \vec{k}_r, w_r)$, $(\vec{E}_t^0, \vec{k}_t, w_t)$

总参数 7　　自由参数 5 (不预设 $k_r^y = 0$, $k_t^y = 0$)

已知&未知参数的优化

引入入射光和反射光的随动二维坐标系，该坐标系垂直于相应的波矢，由坐标轴 $(\hat{s}, \hat{p}_{i/r/t})$ 张成



s : senkrecht (垂直)

p : parallel (平行)

$$\hat{p} \times \hat{s} = \hat{k}$$

自由参数的选取 {

入射光：$(w_i, \theta_i, E_i^{0y}, E_i^{0p})$

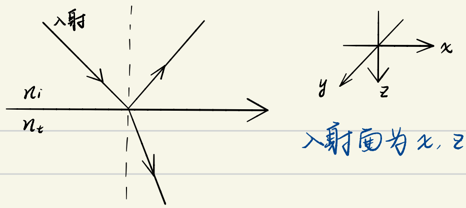
$$\Rightarrow \left(w_i,\ k_i^x = \frac{w_i n_i}{c}\sin\theta_i,\ k_i^z = \frac{w_i n_i}{c}\cos\theta_i,\ E_i^{0x} = E_i^P\cos\theta_i,\ E_i^{0y},\ E_i^{0z} = -E_i^P\sin\theta_i\right)$$

反射光：$(w_r, \theta_r, E_r^{0y}, E_r^{0p}, k_r^y)$

$$\Rightarrow \left(w_r,\ k_r^x = \frac{w_r n_r}{c}\sin\theta_r,\ k_r^y = 0,\ k_r^z = -\frac{w_r n_r}{c}\cos\theta_r,\ E_r^{0x} = -E_r^P\cos\theta_r,\ E_r^{0y},\ E_r^{0z} = -E_r^P\sin\theta_r\right)$$

折射光：$(w_t, \theta_t, E_t^{0y}, E_t^{0p}, k_t^y)$

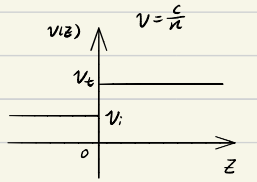
$$\Rightarrow \left(w_t,\ k_t^x = \frac{w_t n_t}{c}\sin\theta_t,\ k_t^y = 0,\ k_t^z = \frac{w_t n_t}{c}\cos\theta_t,\ E_t^{0x} = E_t^P\cos\theta_t,\ E_t^{0y},\ E_t^{0z} = -E_t^P\sin\theta_t\right)$$

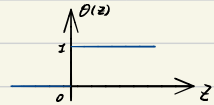


入射面为 x, z 平面 ($k_i^y = 0$)

波动方程

对偶场	$\vec{E}$ & $\vec{B}$
耦合关系	Maxwell 方程
解耦合	$\partial_t^2 \vec{E} = v^2(z)\nabla^2\vec{E}$ + 边界条件
光源	入射光
波动方程	$\partial_t^2 \vec{E} = v^2(z)\nabla^2\vec{E} + \vec{E}_i \cdot \theta(-z)$
1)	当 $z<0$ 时 $\partial_t^2 \vec{E} = \frac{c^2}{n_i^2}\nabla^2\vec{E} + \vec{E}_i$
2)	当 $z>0$ 时 $\partial_t^2 \vec{E} = \frac{c^2}{n_t^2}\nabla^2\vec{E}$
3)	当 $z=0$ 时 $\begin{cases} \vec{E}_{x\text{-}y}(z=0^+) = \vec{E}_{x\text{-}y}(z=0^-) \\ \vec{B}_{x\text{-}y}(z=0^+) = \vec{B}_{x\text{-}y}(z=0^-) \end{cases}$



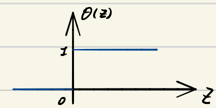


电场切向连续

$\mu_i = \mu_t = 1$

波函数

预解式	$\vec{E} = (\vec{E}_i + \vec{E}_r)\cdot\theta(-z) + \vec{E}_t\cdot\theta(z)$
代入波动方程	验证解的形式，求解解的参数



$z<0$ 预解式成立，且 $\frac{\omega_r}{|k_r|} = \frac{c}{n_r}$

$z>0$ 预解式成立，且 $\frac{\omega_t}{|k_t|} = \frac{c}{n_t}$

$z=0$ $\begin{cases} \vec{E}_{x\text{-}y}(z=0^-) = \vec{E}_{x\text{-}y}(z=0^+) \Rightarrow \vec{E}_i^{x\text{-}y}(z=0) + \vec{E}_r^{x\text{-}y}(z=0) = \vec{E}_t^{x\text{-}y}(z=0) \\ \vec{B}_{x\text{-}y}(z=0^-) = \vec{B}_{x\text{-}y}(z=0^+) \Rightarrow \vec{B}_i^{x\text{-}y}(z=0) + \vec{B}_r^{x\text{-}y}(z=0) = \vec{B}_t^{x\text{-}y}(z=0) \end{cases}$

$\Downarrow$

$$\begin{cases} \omega_i = \omega_r = \omega_t & \text{同频、同色} \\ k_i^y = k_r^y = k_t^y = 0 & \text{共面} \\ k_i^x = k_r^x = k_r^x & \text{波矢匹配条件} \\ E_i^x + E_r^x = E_t^x,\ E_i^y + E_r^y = E_t^y \\ B_i^x + B_r^x = B_t^x,\ B_i^y + B_r^y = B_t^y \end{cases}$$

物理性质（方向、强度）

波的传播方向

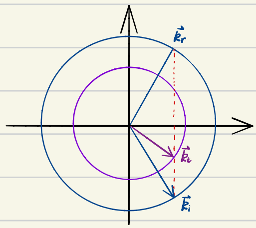
$$\omega_i = \omega_r = \omega_t \qquad k_i^y = k_r^y = k_t^y = 0 \qquad k_i^x = k_r^x = k_t^x$$

Snell's Law

$$\begin{cases} \omega_i = \omega_r = \omega_t \Rightarrow \dfrac{|k_i|}{n_i} = \dfrac{|k_r|}{n_r} = \dfrac{|k_t|}{n_t} \\ k_i^x = k_r^x = k_t^x \Rightarrow |k_i|\sin\theta_i = |k_r|\sin\theta_r = |k_t|\sin\theta_t \end{cases}$$

$$\Rightarrow \begin{cases} n_i\sin\theta_i = n_r\sin\theta_r \Rightarrow \theta_i = \theta_r \quad \text{(折射)} \\ n_i\sin\theta_i = n_t\sin\theta_t \Rightarrow \text{Snell's Law} \quad \text{(反射)} \end{cases}$$

等波矢面法



$$|k_i| = |k_r| = \frac{\omega_i n_i}{c}$$

$$|k_t| = \frac{\omega_t n_t}{c}$$

全反射

等波面法表示

当 $|k_i|\sin\theta_i > |k_t| \Rightarrow \sin\theta_i > \dfrac{|k_t|}{|k_i|} = \dfrac{n_t}{n_i}$ 时发生全反射

解析表示

$$\vec{E}_t = \vec{E}_t^0 e^{i(k_t^x x + k_t^y y - \omega_t t)}$$

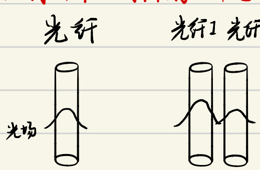
$$\left.\begin{aligned} k_t^x &= k_i^x \\ (k_t^x)^2 + (k_t^z)^2 &= \frac{\omega_t^2 n_t^2}{c^2} \end{aligned}\right\} \Rightarrow k_t^z = \sqrt{\frac{\omega_t^2 n_t^2}{c^2} - (k_i^x)^2}$$

当 $k_i^x > |k_i| = \dfrac{\omega_t n_t}{c}$ 时，k_t^z 为虚数

波函数变为 $\vec{E}_t = \vec{E}_t^0 e^{i(k_t^x x - \omega t)} e^{-\sqrt{(k_i^x)^2 - \frac{\omega_t^2 n_t^2}{c^2}}\, z}$ 沿z方向e指数衰减

△ 全反射并非没有折射，而是折射光的强度沿z轴指数衰减

受阻的全反射



光纤1中的光向光纤2中泄漏

光的强度

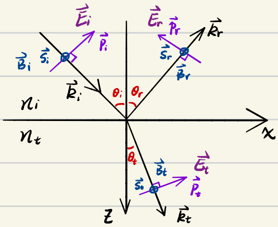
原则

Fresnell's Law 菲涅尔定律

利用电场和磁场的边界条件

电场 $\begin{cases} \vec{n}\times(\vec{E}_1-\vec{E}_2)=0 \\ \vec{n}\cdot(\vec{D}_1-\vec{D}_2)=\rho_f \end{cases}$ 磁场 $\begin{cases} \vec{n}\cdot(\vec{B}_1-\vec{B}_2)=0 \\ \vec{n}\times(\vec{H}_1-\vec{H}_2)=K_f \end{cases}$

特殊情形①



TM mode $\vec{B}=(0,B_y,0)$ $\vec{E}=|E|\cdot\hat{p}$

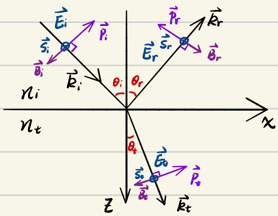
边界条件：$\begin{cases} E_i^x+E_r^x=E_t^x \Rightarrow |E_i|\cos\theta_i-|E_r|\cos\theta_r=|E_t|\cos\theta_t \\ B_i^y+B_r^y=B_t^y \Rightarrow |B_i|+|B_r|=|B_t| \end{cases}$

E-B关系：$|B_i|=\dfrac{|E_i|}{v_i}=\dfrac{|E_i|n_i}{c}$ $|B_r|=\dfrac{|E_r|n_i}{c}$ $|B_t|=\dfrac{|E_t|n_t}{c}$

Snell定律：$n_i\sin\theta_i=n_t\sin\theta_t$ $\theta_i=\theta_t$

$$\Rightarrow \frac{|E_r|}{|E_i|}=\frac{n_t\cos\theta_i-n_i\cos\theta_t}{n_t\cos\theta_i+n_i\cos\theta_t} \qquad \frac{|E_t|}{|E_i|}=\frac{2n_i\cos\theta_i}{n_t\cos\theta_i+n_i\cos\theta_t}$$

特殊情形②



TE mode $\vec{E}=(0,E_y,0)$ $\vec{B}=|B|\cdot(-\hat{p})$

边界条件 $(\mu\approx1)$ $\begin{cases} E_i^y+E_r^y=E_t^y \Rightarrow |E_i|+|E_r|=|E_t| \\ B_i^x+B_r^x=B_t^x \Rightarrow -|B_i|\cos\theta_i+|B_r|\cos\theta_r=-|B_t|\cos\theta_t \end{cases}$

E-B关系：$|B_i|=\dfrac{|E_i|}{v_i}=\dfrac{|E_i|n_i}{c}$ $|B_r|=\dfrac{|E_r|n_i}{c}$ $|B_t|=\dfrac{|E_t|n_t}{c}$

Snell定律：$n_i\sin\theta_i=n_t\sin\theta_t$ $\theta_i=\theta_t$

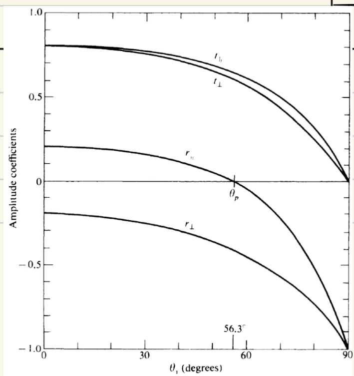
$$\Rightarrow \frac{|E_r|}{|E_i|}=\frac{n_i\cos\theta_i-n_i\cos\theta_t}{n_i\cos\theta_i+n_t\cos\theta_t} \qquad \frac{|E_t|}{|E_i|}=\frac{2n_i\cos\theta_i}{n_i\cos\theta_i+n_t\cos\theta_t}$$

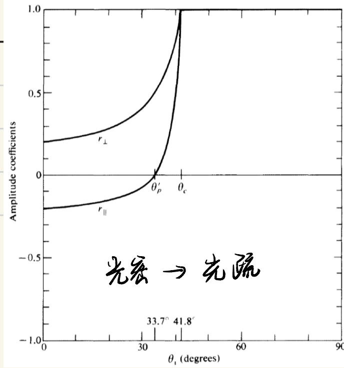
小结

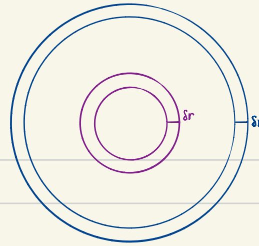
令 $m=\dfrac{\cos\theta_t}{\sin\theta_i}$ $\rho=\dfrac{n_t}{n_i}$

	TM（$\hat{p}$分量）	TE（$\hat{s}$分量）
振幅反射率	$r_p=\dfrac{\rho-m}{\rho+m}$	$r_s=\dfrac{1-\rho m}{1+\rho m}$
振幅透射率	$t_p=\dfrac{2}{\rho+m}$	$t_s=\dfrac{2}{1+\rho m}$

光疏 → 光密





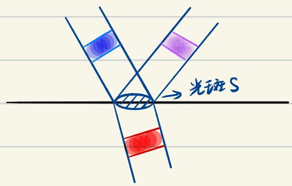


能量守恒

物理背景

在t时刻空间某一体积内的光能传播到 $t+\delta t$ 后，能量不变

物理系统



t时刻的体积

$t+\delta t$ 时刻在反射光

$t+\delta t$ 时刻在折射光

求解

① 求 $V(t)$ 和 $V(t+\delta t)$

$$V_i(t) = S_i h_i = S h_i \cos\theta_i$$

$$V_r(t+\delta t) = S_r h_r = S h_r \cos\theta_r$$

$$V_t(t+\delta t) = S_t h_t = S h_t \cos\theta_t$$

$$\begin{cases} \dfrac{h_i}{v_i} = \dfrac{h_r}{v_r} = \dfrac{h_t}{v_t} \\ v = \dfrac{c}{n} \end{cases}$$

$$\Rightarrow n_i h_i = n_i h_r = n_t h_t$$

② 求电磁场能量密度

$$\rho_i = \frac{\varepsilon_i|E_i|^2}{2} + \frac{|B_i|^2}{2} = \varepsilon_i|E_i|^2 = \frac{n_i^2}{c^2}|E_i|^2$$

$$\rho_r = \varepsilon_i|E_r|^2 = \frac{n_i^2}{c^2}|E_r|^2 = \frac{n_i^2}{c^2} r^2 |E_i|^2$$

$$\rho_t = \varepsilon_t|E_t|^2 = \frac{n_t^2}{c^2}|E_t|^2 = \frac{n_t^2}{c^2} t^2 |E_i|^2$$

③ 验证 $U_i(t) = U_r(t+\delta t) + U_t(t+\delta t)$

$$U_i(t) = \rho_i V_i(t) = \frac{n_i^2}{c^2}|E_i|^2 \cdot S h_i \cos\theta_i = \frac{n_i h_i}{c^2} S|E_i|^2 \cdot n_i \cos\theta_i = I_i \cos\theta_i$$

$$U_r(t+\delta t) = \rho_r V_r(t+\delta t) = \frac{n_i^2}{c^2} r^2 |E_i|^2 \cdot S h_r \cos\theta_r = \frac{n_i h_i}{c^2} S|E_i|^2 \cdot r^2 n_i \cos\theta_i = I_r \cos\theta_i$$

$$U_t(t+\delta t) = \rho_t V_t(t+\delta t) = \frac{n_t^2}{c^2} t^2 |E_i|^2 \cdot S h_t \cos\theta_t = \frac{n_i h_i}{c^2} S|E_i|^2 \cdot t^2 n_t \cos\theta_t = I_t \cos\theta_t$$

$$I_i \cos\theta_i = I_r \cos\theta_r + I_t \cos\theta_t$$

强度反射率 $R = \dfrac{I_r}{I_i} = r^2$

强度透射率 $T = \dfrac{I_t}{I_i} = \dfrac{n_t}{n_i} t^2$

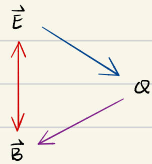
表 6－1 各种反射率和透射率的定义

	p 分量	s 分量
振幅反射率	$\tilde{r}_p = \tilde{E}'_{1p}/\tilde{E}_{1p}$ (6.13)	$\tilde{r}_s = \tilde{E}'_{1s}/\tilde{E}_{1s}$ (6.14)
强度反射率	$R_p = \dfrac{I'_{1p}}{I_{1p}} = \lvert\tilde{r}_p\rvert^2$ (6.15)	$R_s = \dfrac{I'_{1s}}{I_{1s}} = \lvert\tilde{r}_s\rvert^2$ (6.16)
能流反射率	$\mathscr{R}_p = \dfrac{W'_{1p}}{W_{1p}} = R_p$ (6.17)	$\mathscr{R}_s = \dfrac{W'_{1s}}{W_{1s}} = R_s$ (6.18)
振幅透射率	$\tilde{t}_p = \tilde{E}_{2p}/\tilde{E}_{1p}$ (6.19)	$\tilde{t}_s = \tilde{E}_{2s}/\tilde{E}_{1s}$ (6.20)
强度透射率	$T_p = \dfrac{I_{2p}}{I_{1p}} = \dfrac{n_2}{n_1}\lvert\tilde{t}_p\rvert^2$ (6.21)	$T_s = \dfrac{I_{2s}}{I_{1s}} = \dfrac{n_2}{n_1}\lvert\tilde{t}_s\rvert^2$ (6.22)
能流透射率	$\mathscr{T}_p = \dfrac{W_{2p}}{W_{1p}} = \dfrac{\cos i_2}{\cos i_1} T_p$ (6.23)	$\mathscr{T}_s = \dfrac{W_{2s}}{W_{1s}} = \dfrac{\cos i_2}{\cos i_1} T_s$ (6.24)

金属的折射和反射

波动方程

对偶场



M3 $\nabla\times\vec E = -\frac{\partial \vec B}{\partial t}$

欧姆定律 $\vec J = \sigma\vec E$

M4 $\nabla\times\vec B = \mu\left(\vec J + \varepsilon\frac{\partial \vec E}{\partial t}\right)$

解耦合

$$\left.\begin{aligned}\nabla\times\vec E &= -\frac{\partial \vec B}{\partial t}\\ \nabla\times\vec B &= \mu\left(\vec J + \varepsilon\frac{\partial \vec E}{\partial t}\right)\end{aligned}\right\} \Rightarrow \left.\begin{aligned}\varepsilon\mu\partial_t^2\vec E &= \nabla^2\vec E - \mu\partial_t\vec J\\ \vec J &= \sigma\vec E\end{aligned}\right\}$$

$$\Rightarrow \varepsilon\mu\partial_t^2\vec E = \nabla^2\vec E - \underbrace{\mu\sigma\partial_t\vec E}_{}$$ 耗散项

求解波函数

试探解 $E = E_0 e^{i(\vec k\cdot\vec r - \omega t)}$

代入波动方程 $-\omega^2\varepsilon\mu\vec E = -k^2\vec E + \mu\sigma(i\omega)\vec E$

$$\Rightarrow k^2 = \omega^2\left(\varepsilon\mu + \frac{i\mu\sigma}{\omega}\right)$$

$$\Rightarrow n^2 = \frac{c^2}{v^2} = c^2\frac{k^2}{\omega^2} = 1 + \frac{i\sigma}{\varepsilon\omega}$$

物理性质

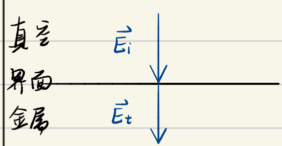
金属中的相速度

$$v_p = \frac{\omega}{|k|} = \sqrt{\varepsilon\mu + \frac{i\mu\sigma}{\omega}}$$

金属中的折射率

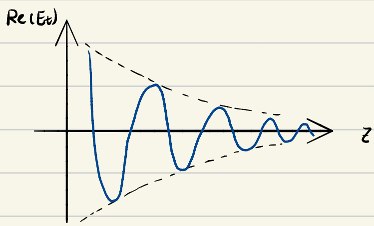
$$n = \sqrt{1 + \frac{i\sigma}{\varepsilon\omega}}$$

趋肤深度



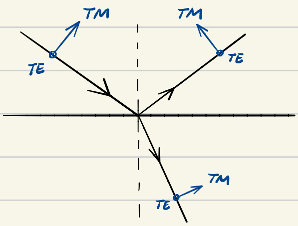
垂直入射时 $E_t = E_0 e^{i\overset{\text{复数}}{k_R} z} e^{-k_I z}$

$I = I_0 e^{-2k_I z}$



$$\delta = \frac{1}{2}k_I = \frac{1}{\sqrt{2\omega\mu_0\sigma}}$$

折射&反射回顾



琼斯矢量 & 矩阵的描述

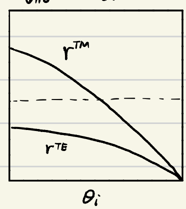
- 入、反、折射光的 J-矢量 $\begin{bmatrix} E_i^{TM} \\ E_i^{TE} \end{bmatrix}_{入} \oplus \begin{bmatrix} E_r^{TM} \\ E_r^{TE} \end{bmatrix}_{反} \oplus \begin{bmatrix} E_t^{TM} \\ E_t^{TE} \end{bmatrix}_{透}$
- 折射&反射：J-矩阵

$$\begin{bmatrix} E_r^{TM} \\ E_r^{TE} \end{bmatrix} = \begin{bmatrix} r_{TM} & 0 \\ 0 & r_{TE} \end{bmatrix} \begin{bmatrix} E_i^{TM} \\ E_i^{TE} \end{bmatrix} \qquad \begin{bmatrix} E_t^{TM} \\ E_t^{TE} \end{bmatrix} = \begin{bmatrix} t_{TM} & 0 \\ 0 & t_{TE} \end{bmatrix} \begin{bmatrix} E_i^{TM} \\ E_i^{TE} \end{bmatrix}$$

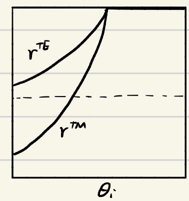
S-定律 & F-公式

- F-公式

疏 ⇒ 密



密 ⇒ 疏



- S-定律 & F-公式适用所有平面波情形

折射&反射 ⇒ 偏振

概括

① 利用B角入射产生/检测偏振光 ⇒ 偏振光

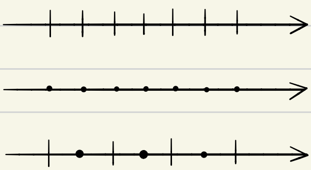
② 利用全反射调节偏振光 ⇒ 波片

利用B-角(布鲁斯特角)做偏振片

B-角入射的回顾

定性描述
- B-角入射 ⇒ { 反射光是纯TE光；透射光是TM+TE光
- 琼斯矢量J-m $\begin{bmatrix} E_i^{TM} \\ E_i^{TE} \end{bmatrix}_{入} \Rightarrow \begin{bmatrix} 0 \\ rE_i^{TE} \end{bmatrix}_{反} + \begin{bmatrix} t_{TM}E_i^{TM} \\ t_{TE}E_i^{TE} \end{bmatrix}_{透}$
- $\overleftrightarrow{J}_{反射} = \begin{bmatrix} 0 & 0 \\ 0 & r \end{bmatrix} \qquad \overleftrightarrow{J}_{透射} = \begin{bmatrix} t_{TM} & 0 \\ 0 & t_{TE} \end{bmatrix}$

画图约定

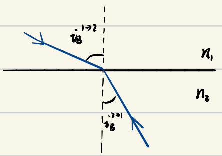


TM

TE

TM + TE

定量描述（计算 i_B）



- 出发点：Snell定律 & Fresnel公式
- 关系① $r_{TM} = r_{\parallel} = \frac{p-m}{p+m} = 0 \Rightarrow p = m$

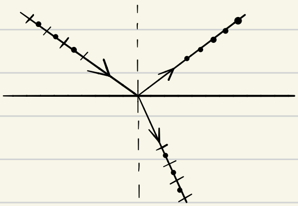
  $p = \frac{n_t}{n_i} \qquad m = \frac{\cos\theta_t}{\cos\theta_i}$

  $\Rightarrow \frac{\cos\theta_t}{\cos\theta_i} = \frac{n_t}{n_i} = \frac{\sin\theta_i}{\sin\theta_t} \Rightarrow \sin\theta_t\cos\theta_t = \sin\theta_i\sin\theta_t$

  $\Rightarrow \sin2\theta_t = \sin2\theta_i \Rightarrow \theta_t = \frac{\pi}{2} - \theta_i = \frac{\pi}{2} - i_B$

  B-角入射时，入射角和折射角互余

- 关系② $\sin^2 i_B = \frac{n_t^2}{n_i^2+n_t^2} \Rightarrow i_B^{2\to1} + i_B^{1\to2} = \frac{\pi}{2}$

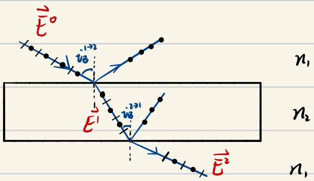


单层界面

$\overleftrightarrow{J}_{反} = \boxed{\begin{bmatrix}0 & 0\\ 0 & r_{TE}\end{bmatrix}} \qquad \overleftrightarrow{J}_{透} = \begin{bmatrix}t_{TM} & 0\\ 0 & t_{TE}\end{bmatrix}$

偏振片

双层界面



描述 { 从 $n_1 \to n_2$ 以 $i_B^{1\to2}$ 入射时，在第二个界面入射角恰等于 $i_B^{2\to1}$
经过两次折射，透射光中TE模式进一步减少

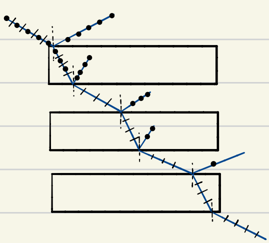
$\vec{E}^2 = \overleftrightarrow{J}^{(2)}_{透}\vec{E}^1 = \overleftrightarrow{J}^{(2)}_{透}\overleftrightarrow{J}^{(1)}_{透}\vec{E}^0$ 证明

$\Rightarrow \overleftrightarrow{J}^{(1)+(2)}_{透} = \overleftrightarrow{J}^{(2)}_{透}\cdot\overleftrightarrow{J}^{(1)}_{透}$ （类似ABCD矩阵）

$$\overleftrightarrow{J}^{(1)+(2)}_{透} = \begin{bmatrix}t_\parallel^{2\to1} & 0\\ 0 & t_\perp^{2\to1}\end{bmatrix}\begin{bmatrix}t_\parallel^{1\to2} & 0\\ 0 & t_\perp^{1\to2}\end{bmatrix} = \begin{bmatrix}t_\parallel^{2\to1}t_\parallel^{1\to2} & 0\\ 0 & \underbrace{t_\perp^{2\to1}t_\perp^{1\to2}}_{\text{小于单层膜}}\end{bmatrix}$$

$t_\perp^{2\to1}\cdot t_\perp^{1\to2} \ll t_\perp^{1\to2}$ 经过两层膜之后，透射光中的TE模式 ↓↓

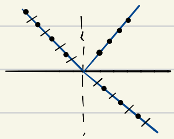
多层膜



经过多层膜B-角反射&折射

反射光纯TE　　透射光纯TM

$J^{多层}_{反} = \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix} \qquad J^{多层}_{透} = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$

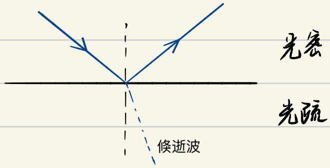


小结

$$J_{反} = \begin{bmatrix} 0 & 0 \\ 0 & r_\perp \end{bmatrix} \qquad J_{透} = \begin{bmatrix} t_{\parallel} & 0 \\ 0 & t_\perp \end{bmatrix}$$

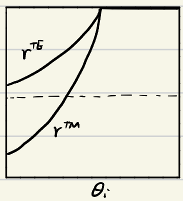
入射光	单层		多层	
	t	r	t	r
线偏	线	线(TE)	线(TM)	线(TE)
圆偏	椭			
椭偏	椭			
自然光	部分偏振			

利用全反射做波片



全反射回顾

密 ⇒ 疏



全反射时 $|r_{\parallel}| = |r_\perp| = 1$ $\quad r_{\parallel} = e^{i\phi_{\parallel}}$ $\quad r_\perp = e^{i\phi_\perp}$

证明：S-定律 ⇒ $n_t \sin\theta_t = n_i \sin\theta_i$ $\qquad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

发生全反射时 $\sin\theta_t = \frac{n_i \sin\theta_i}{n_t} > 1$ (θ_t 是复数)

F-公式 ⇒ $r_{\parallel} = \frac{p-m}{p+m}$ 可证 $r_{\parallel} r_{\parallel}^* = 1$ $\quad |r_{\parallel}| = 1$

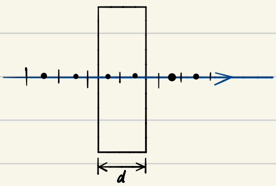
同理 $|r_\perp| = 1$ 且 $r_{\parallel}$ 和 $r_\perp$ 为复数

$r_{\parallel} = e^{i\phi_{\parallel}} \qquad r_\perp = e^{i\phi_\perp}$

偏振调节

- $$\overleftrightarrow{J}_r = \begin{bmatrix} r_{\parallel} & 0 \\ 0 & r_\perp \end{bmatrix} \xlongequal{\text{全反射}} \begin{bmatrix} e^{i\phi_{\parallel}} & 0 \\ 0 & e^{i\phi_\perp} \end{bmatrix} = e^{i\phi_{\parallel}} \begin{bmatrix} 1 & 0 \\ 0 & e^{i(\phi_\perp - \phi_{\parallel})} \end{bmatrix}$$
  $$= e^{i\phi_{\parallel}} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\Delta\phi} \end{bmatrix}$$
- TM和TE模式的光在全反射时，有不同的相位效果（波片）
- $$\begin{bmatrix} E_r^{TM} \\ E_r^{TE} \end{bmatrix} = \overleftrightarrow{J}_r \begin{bmatrix} E_i^{TM} \\ E_i^{TE} \end{bmatrix} = e^{i\phi_{\parallel}} \begin{bmatrix} E_i^{TM} \\ e^{i\Delta\phi} E_i^{TE} \end{bmatrix}$$

$$J_{波} = e^{i\phi}\begin{bmatrix} 1 & 0 \\ 0 & e^{i\Delta\phi} \end{bmatrix}$$



波片

- 功能：对两个互相垂直的偏振方向，诱导不同的相位延迟
- 特征指标：快轴

  对于TM、TE模式的光，相位积累(光程)少的轴是快轴

  考虑一个波片，波片中 $v_{TE} > v_{TM} \Rightarrow k_{TE} < k_{TM}$ $\left(v = \frac{\omega}{k}\right)$

  光程 $\phi_{TE} = k_{TE} d < k_{TM} d = \phi_{TM}$

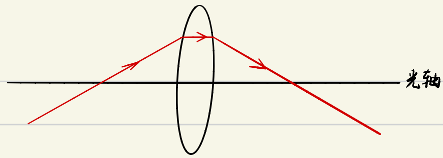
  e.g. 若快轴为TE轴，则 $\phi_{TM} > \phi_{TE}$ $\Delta\phi = \phi_{TE} - \phi_{TM} < 0$

  $$\overleftrightarrow{J} = e^{i\phi}\begin{bmatrix} 1 & 0 \\ 0 & e^{i(\phi_{TE}-\phi_{TM})} \end{bmatrix} = e^{i\phi}\begin{bmatrix} 1 & 0 \\ 0 & e^{i\Delta\phi} \end{bmatrix}$$

- example

入射光	$\frac{1}{8}$波片 ~ $\Delta\phi = \frac{2\pi}{8} = \frac{\pi}{4}$	$\frac{1}{4}$波片 ~ $\Delta\phi = \frac{2\pi}{4} = \frac{\pi}{2}$
线偏 $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$	椭偏 $\begin{bmatrix} 1 \\ 2e^{i\frac{\pi}{4}} \end{bmatrix}$ (一般)	椭偏 $\begin{bmatrix} 1 \\ 2e^{i\frac{\pi}{2}} \end{bmatrix}$ (一般)
圆偏 $\begin{bmatrix} 1 \\ i \end{bmatrix}$	椭偏 $\begin{bmatrix} 1 \\ e^{i\frac{3\pi}{4}} \end{bmatrix}$	线偏 $\begin{bmatrix} 1 \\ e^{\pm i\pi} \end{bmatrix} \sim \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
椭偏	椭偏	椭偏
自然	自然	自然

几何光学

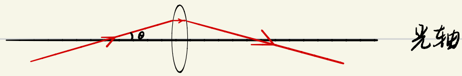


物理系统

光：光沿直线传播

Snell's Law $\begin{cases} \theta_i = \theta_r \\ n_i \sin\theta_i = n_t \sin\theta_t \end{cases}$

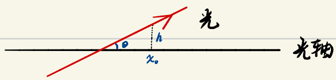
光学元器件：存在一个对称轴 $\Rightarrow$ 光轴



傍轴近似：$\theta \approx 0^\circ$

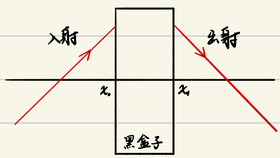
数学建模

光的建模：用最简单的一串数字完整地描述光线



$\Rightarrow (h, \theta)_{x_0} \Rightarrow \begin{pmatrix} h \\ \theta \end{pmatrix}_{x_0}$

光学元器件



原则：光学元器件的作用是把一束入射光变成一束出射光

$\Leftrightarrow$ 建立一个从入射光 $\begin{pmatrix} h_入 \\ \theta_入 \end{pmatrix}_{x_0}$ 到出射光 $\begin{pmatrix} h_出 \\ \theta_出 \end{pmatrix}_{x_1}$ 的函数

数学模型：

$$\begin{cases} h_出 = f_1(h_入, \theta_入) \\ \theta_出 = f_2(h_入, \theta_入) \end{cases} \xrightarrow[f_1, f_2 是线性函数]{线性近似} \begin{cases} h_出 = a h_入 + b\theta_入 \\ \theta_出 = c h_入 + d\theta_入 \end{cases}$$

ABCD矩阵 $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$：光学元器件由 (a, b, c, d) 4个数字确定

$$\begin{bmatrix} h_出 \\ \theta_出 \end{bmatrix}_{x_1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} h_入 \\ \theta_入 \end{bmatrix}_{x_0}$$

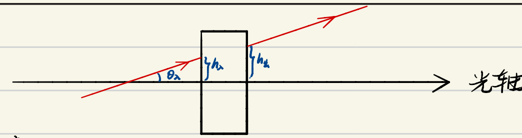
傍轴近似：线性近似 (f_1, f_2)

光学元器件举例

单个光学元器件

真空

$$\begin{bmatrix} 1 & x_1-x_0 \\ 0 & 1 \end{bmatrix}$$

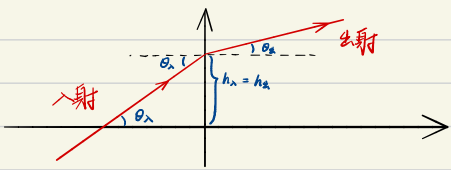


最简单的光学元器件

$$\begin{cases} h_出 = h_入 + (x_1-x_0)\tan\theta_入 \\ \theta_出 = \theta_入 \end{cases} \xrightarrow[\tan\theta_入 \sim \theta_入]{傍轴近似} \begin{cases} h_出 = h_入 + (x_1-x_0)\theta_入 \\ \theta_出 = 0\cdot h_入 + \theta_入 \end{cases}$$

折射界面

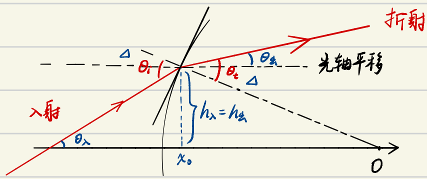
$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{n_入}{n_出} \end{bmatrix}$$



$$\begin{cases} h_出 = h_入 \\ n_入\sin\theta_入 = n_出\sin\theta_出 \end{cases} \xrightarrow{傍轴近似} \begin{cases} h_出 = h_入 + 0\cdot\theta_入 \\ \theta_出 = 0\cdot h_入 + \frac{n_入}{n_出}\theta_入 \end{cases}$$

球形界面

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n_入}{n_出}-1\right) & \frac{n_入}{n_出} \end{bmatrix}$$

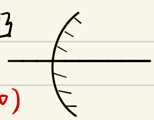


曲面曲率很小

$\Rightarrow R \gg$ 所有特征尺寸

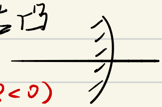
$$\begin{cases} h_出 = h_入 \\ \begin{cases} \theta_i = \theta_入 + \Delta \\ \theta_t = \theta_出 + \Delta \\ \sin\Delta = \frac{h_入}{R} \xrightarrow{曲率小} \Delta = \frac{h_入}{R} \\ n_入\sin\theta_i = n_出\sin\theta_t \xrightarrow{傍轴似近} n_入\theta_i = n_出\theta_t \end{cases} \end{cases} \Longrightarrow \begin{cases} h_出 = h_入 + 0\cdot\theta_入 \\ \theta_出 = \frac{1}{R}\left(\frac{n_入}{n_出}-1\right)h_入 + \frac{n_入}{n_出}\theta_入 \end{cases}$$

左凹 $(R>0)$



$$\begin{bmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n_i}{n_0}-1\right) & \frac{n_i}{n_0} \end{bmatrix}$$

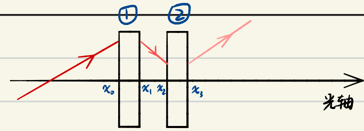
右凹 $(R<0)$



$$\begin{bmatrix} 1 & 0 \\ \frac{1}{|R|}\left(1-\frac{n_i}{n_0}\right) & \frac{n_i}{n_0} \end{bmatrix}$$

组合光学元器件(共轴)

抽象系统



$$\begin{bmatrix} a_{12} & b_{12} \\ c_{12} & d_{12} \end{bmatrix} = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \otimes \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

推导

入 $\begin{bmatrix} h_{入} \\ \theta_{入} \end{bmatrix}$ 中 $\begin{bmatrix} h_1 \\ \theta_1 \end{bmatrix}_{x_1} \rightarrow \begin{bmatrix} h_2 \\ \theta_2 \end{bmatrix}_{x_2}$ 出 $\begin{bmatrix} h_{出} \\ \theta_{出} \end{bmatrix}$

$$\begin{bmatrix} h_{出} \\ \theta_{出} \end{bmatrix}_{x_3} = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \begin{bmatrix} h_2 \\ \theta_2 \end{bmatrix}_{x_2}$$

$$\begin{bmatrix} h_2 \\ \theta_2 \end{bmatrix}_{x_2} = \begin{bmatrix} 1 & x_2 - x_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ \theta_1 \end{bmatrix}_{x_1}$$

$$\begin{bmatrix} h_1 \\ \theta_1 \end{bmatrix}_{x_1} = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \begin{bmatrix} h_{入} \\ \theta_{入} \end{bmatrix}_{x_0}$$

$$\Rightarrow \begin{bmatrix} h_{出} \\ \theta_{出} \end{bmatrix}_{x_3} = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \begin{bmatrix} 1 & x_2 - x_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} = \begin{bmatrix} a_{12} & b_{12} \\ c_{12} & d_{12} \end{bmatrix} \begin{bmatrix} h_{入} \\ \theta_{出} \end{bmatrix}$$

光元①+真空+光元②

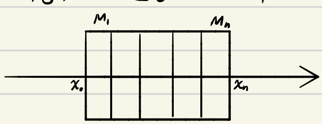
$$\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \begin{bmatrix} 1 & x_2 - x_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

光元①+光元②

$$\lim_{x_1 \to x_2} (\text{光元①} + \text{真空} + \text{光元②}) \Rightarrow \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$$

推广

任意共轴光学元器件的组合

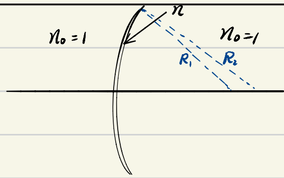


$$M_i \sim \begin{bmatrix} a_i & b_i \\ c_i & d_i \end{bmatrix}$$

整体 $$\begin{bmatrix} a_{total} & b_{total} \\ c_{total} & d_{total} \end{bmatrix} = \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix} \begin{bmatrix} a_{n-1} & b_{n-1} \\ c_{n-1} & d_{n-1} \end{bmatrix} \cdots \begin{bmatrix} a_1 & c_1 \\ b_1 & d_1 \end{bmatrix}$$

举例

薄凹透镜

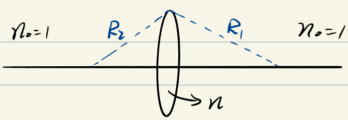


$$M_{透镜} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n_1}{n_2}-1\right) & \frac{n_1}{n_2} \end{bmatrix}_{R_2} \begin{bmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n_1}{n_2}-1\right) & \frac{n_1}{n_2} \end{bmatrix}_{R_1} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R_2}\left(\frac{n}{n_0}-1\right) & \frac{n}{n_0} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_1}\left(\frac{n_0}{n}-1\right) & \frac{n_0}{n} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ \frac{1}{R_2}(n-1) & n \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_1}\left(\frac{1}{n}-1\right) & \frac{1}{n} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ (n-1)\left(\frac{1}{R_2}-\frac{1}{R_1}\right) & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

透镜焦距 f $\quad \frac{1}{f} = (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

薄凸透镜



$$M_{透镜} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R_2}\left(1-\frac{n_1}{n_2}\right) & \frac{n_1}{n_2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_1}\left(\frac{n_1}{n_2}-1\right) & \frac{n_1}{n_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R_2}\left(1-\frac{n}{n_0}\right) & \frac{n}{n_0} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_1}\left(\frac{n_0}{n}-1\right) & \frac{n_0}{n} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ \frac{1}{R_2}(1-n) & n \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{R_1}\left(\frac{1}{n}-1\right) & \frac{1}{n} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ (1-n)\left(\frac{1}{R_1}+\frac{1}{R_2}\right) & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

焦距 f $\quad \frac{1}{f} = (n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

偏振

波函数

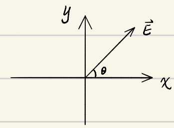
$$\partial_t^2 \vec{E} = \frac{1}{\mu\varepsilon}\nabla^2 \vec{E} \iff \begin{cases} \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \mu\varepsilon\frac{\partial^2 E_x}{\partial t^2} \\ \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu\varepsilon\frac{\partial^2 E_y}{\partial t^2} \\ \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = \mu\varepsilon\frac{\partial^2 E_z}{\partial t^2} \end{cases}$$

(E_x, E_y, E_z) 可以取不同的波函数

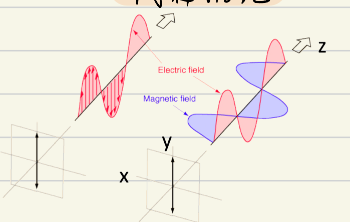
自然光(无偏振光)

各方向无优越性

线偏振光



$$\begin{cases} E_x = E\cos\theta = E_0\cos\theta\, e^{i(kz-wt)} = E_0^x e^{i(kz-wt)} \\ E_y = E\sin\theta = E_0\sin\theta\, e^{i(kz-wt)} = E_0^y e^{i(kz-wt)} \end{cases}$$

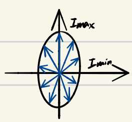


$$\begin{bmatrix} E_x(z,t) \\ E_y(z,t) \end{bmatrix} = \begin{bmatrix} E_0^x \\ E_0^y \end{bmatrix} e^{i(kz-wt)} = E_0 \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} e^{i(kz-wt)} = E_0^x \begin{bmatrix} 1 \\ \tan\theta \end{bmatrix} e^{i(kz-wt)}$$

琼斯矢量 $\sim \left\{ \begin{bmatrix} E_0^x \\ E_0^y \end{bmatrix}; \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}; \begin{bmatrix} 1 \\ \tan\theta \end{bmatrix} \right\}$

部分偏振光

介于自然光和线偏光之间

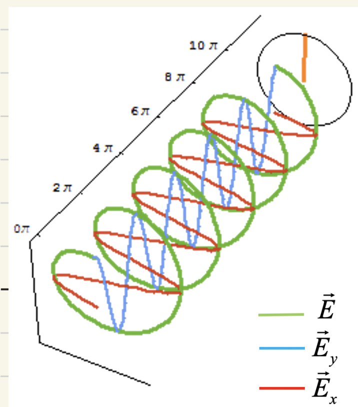


偏振度 $P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

$P = 0$ (自然光/非偏振光)

$P = 1$ (线偏振光)

圆偏振光

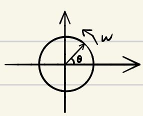


$$\begin{cases} E_x = E_0\cos(kz-wt) = \mathrm{Re}[E_0 e^{i(kz-wt)}] \\ E_y = E_0\cos(kz-wt \pm \frac{\pi}{2}) = \mathrm{Re}[E_0 e^{i(kz-wt\pm\frac{\pi}{2})}] \end{cases}$$

$$\begin{bmatrix} E_x(z,t) \\ E_y(z,t) \end{bmatrix} = E_0 \begin{bmatrix} 1 \\ e^{\pm i\frac{\pi}{2}} \end{bmatrix} e^{i(kz-wt)}$$

琼斯矢量

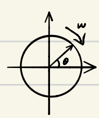
逆时针(左旋圆偏振光)



$$\begin{cases} E_x = E_0\cos(kz-wt) \\ E_y = E_0\sin(kz-wt) \end{cases}$$

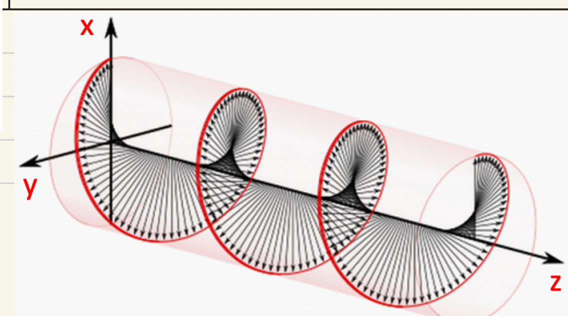
$= E_0\cos(kz-wt-\frac{\pi}{2})$

顺时针(右旋圆偏振光)

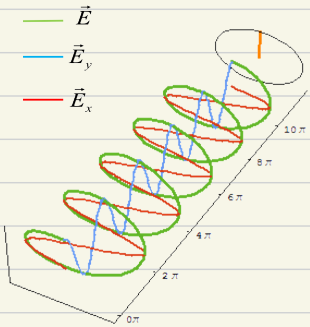


$$\begin{cases} E_x = E_0\cos(kz-wt) \\ E_y = -E_0\sin(kz-wt) \end{cases}$$

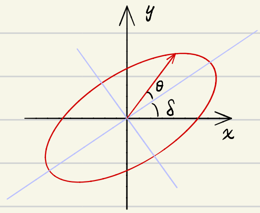
$= E_0\cos(kz-wt+\frac{\pi}{2})$



椭圆偏振光



线偏振光 $\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$ & 圆偏振光 $\begin{bmatrix}1\\ e^{\pm i\frac{\pi}{2}}\end{bmatrix}$



$$\begin{cases} E_x(z,t) = E_0^x e^{i(kz-wt)} = E_0\cos\theta\, e^{i(kz-wt)} \\ E_y(z,t) = E_0^y e^{i(kz-wt+\delta)} = E_0\sin\theta\, e^{i(kz-wt+\delta)} \end{cases}$$

可看成是两个相互垂直的简谐运动的合成

但振幅不等或相位差不等于 $\pm\frac{\pi}{2}$

琼斯矢量

$$\begin{bmatrix}E_x(z,t)\\ E_y(z,t)\end{bmatrix} = \begin{bmatrix}E_0^x\\ E_0^y e^{i\delta}\end{bmatrix} e^{i(kz-wt)} = E_0 \begin{bmatrix}\cos\theta\\ \sin\theta\, e^{i\delta}\end{bmatrix} e^{i(kz-wt)}$$

$\delta=\pi$	δ位于第三象限	$\delta=-\frac{\pi}{2}$	δ位于第四象限	$\delta=0$
$\delta=0$	δ位于第一象限	$\delta=\frac{\pi}{2}$	δ位于第二象限	$\delta=\pi$

推广

框架

↓

遍历

↓

打破框架

从框架遍历到打破框架

$$\begin{bmatrix}E_x(z,t)\\ E_y(z,t)\end{bmatrix} = \begin{bmatrix}E_0^x\\ E_0^y\end{bmatrix} e^{i(kz-wt)}$$

$\begin{bmatrix}E_0^x\\ E_0^y\end{bmatrix}$ 取遍任意复数 $\Rightarrow$ 椭偏光

含时振荡 $\Rightarrow$ 自然光

$$\begin{cases} E_x(z,t) \rightarrow E_x(x,y,z,t) \\ E_y(z,t) \rightarrow E_y(x,y,z,t) \end{cases}$$

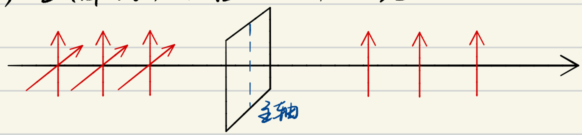
具有轨道角动量的光：LG光束、Gauss光束

偏振的应用

偏振片

作用

产生偏振光，检测偏振光

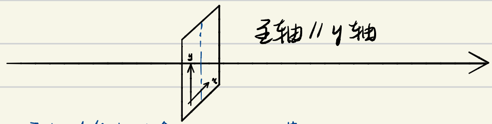


性质

偏振片存在一个主轴，它的作用是过滤掉入射光中电场方向与主轴垂直的光，只透过电场方向（偏振方向）和主轴平行的光。

矢量描述

系统



偏振片的作用

（实数/复数/含时随机振荡）

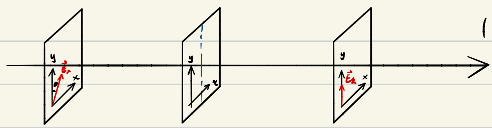
入射光的琼斯矢量 $\begin{bmatrix} E_0^x \\ E_0^y \end{bmatrix} \xRightarrow{\text{偏振片}}$ 出射光 $\begin{bmatrix} 0 \\ E_0^y \end{bmatrix}$

矩阵描述（琼斯矩阵）~ $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ （前提：主轴 // y 轴）

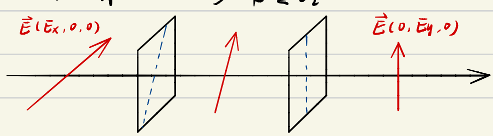
能量视角

马吕斯定律 $I_{出} = A_{出}^2 = (A_{入}\cos\theta)^2 = A_{入}^2\cos^2\theta = I_{入}\cos^2\theta$

（θ 光的偏振方向与主轴夹角）



推广：主轴1和入射光偏振夹角为 θ_1；
主轴2和主轴1夹角是 θ_2



马吕斯定律 $I_2 = I_1\cos^2\theta_2 = I_0\cos^2\theta_1\cos^2\theta_2$

二向色性

基本原则

光在介质中的传播性质，完全由光和介质的α的相互作用决定

极端情况

① 光和介质的α没有相互作用（光不能激发α的运动）

光视介质为真空

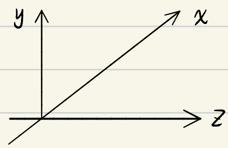
② 光和α有相互作用，且α的损耗 ~ ∞

光不能在介质中传播

二向色性：同一介质中整合了以上两个极端情况

系统

设介质的α是如下情况



各向异性 $\begin{cases} \text{沿 } x \text{ 方向无法振动，无法被激发} \\ \text{沿 } y \text{ 方向有很强的损耗} \end{cases}$

波动方程

$$\vec{E} \leftrightarrow \vec{B},\quad \vec{E} \to \vec{\alpha} \to \vec{B} \quad \xRightarrow{z\text{方向传播的光}} \quad E_x \leftrightarrow B_y,\ E_x \to \alpha_x \to B_y \quad \& \quad E_y \leftrightarrow B_x,\ E_y \to \alpha_y \to B_x$$

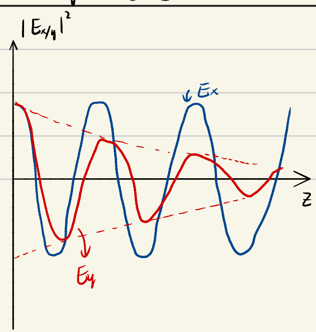
二维偏振波动方程：α振动

$$\begin{cases} \partial_t^2 u_x = \infty \cdot u_x + E_x & \Rightarrow u_x = \dot{u}_x = 0 \\ \partial_t^2 u_y = k u_y + E_y - \gamma_y \dot{u}_y \end{cases}$$

$$\begin{cases} \partial_t^2 E_x = c^2 \partial_z^2 E_x \\ \partial_t^2 E_y = v^2 \partial_z^2 E_y \quad (v = v_r + i v_i) \end{cases}$$

波函数

$$\begin{cases} E_x = E_0^x e^{i(kz-ckt)} \\ E_y = E_0^y e^{i(kz-v_r kt)} e^{-|k_i| z} \end{cases}$$



性质

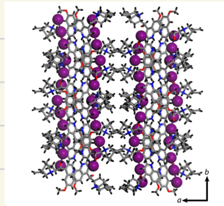
入射光中
- 沿x方向偏振的部分 ⇒ 完美透过
- 沿y方向偏振的部分 ⇒ 完全吸收(过滤)

⇒ 偏振片 主轴沿x方向

真实系统中的实现

自然界中的系统

eg ~ 生物大分子

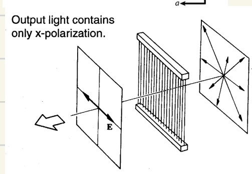


沿a方向: 基本无运动,
不被激发

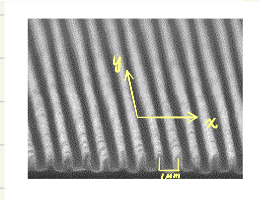
沿b方向: 可运动、有吸收

宏观的例子

eg ~ 人造材料



微观的例子



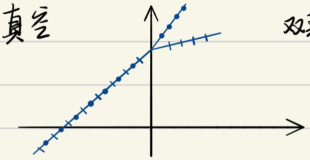
双折射

机制

名词解释

真空 双射折介质

- 物理过程



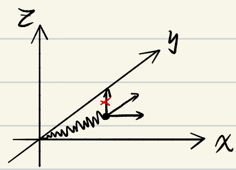
- 语言描述

TE 和 TM 模式的光发生不同的折射现象

物理系统

- 光：平面波
- 介质：有各向异性的 α （e.g. 电偶极子）
- Example：模型特点 ① 电偶极子，类比为弹簧振子

② 有各向异性 $\begin{cases} x\text{-}y \text{ 平面内可振动} \\ \text{沿 } z \text{ 方向不能振动} \end{cases}$



电偶极子的运动方程

$$\begin{cases} m\ddot{u}_x = k u_x + E_x \\ m\ddot{u}_y = k u_y + E_y \\ u_z = \dot{u}_z = 0 \end{cases}$$

波动方程

- 对偶场

$\vec{E} \updownarrow \vec{B}$，$\vec{E} \to \alpha \to \vec{B}$

$\{\vec{E}(\vec{r},t),\ \vec{B}(\vec{r},t),\ \alpha(\vec{r},t)\}$ $\to \vec{p}(\vec{r},t)$

- 耦合关系

○ 库仑定律

$$\begin{cases} m\ddot{u}_x = k u_x + E_x \\ m\ddot{u}_y = k u_y + E_y \\ u_z = \dot{u}_z = 0 \end{cases}$$

○ M4 $\quad \frac{1}{\mu}\vec{\nabla}\times\vec{B} = \varepsilon\partial_t\vec{E} + \vec{J}$

○ M3 $\quad \vec{\nabla}\times\vec{E} = -\partial_t\vec{B}$

- 解耦合

（见下页）

波动方程

- 对偶场
- 耦合关系
- 解耦合

$$\mu_0\begin{bmatrix}\varepsilon_0+\varepsilon_1 & 0 & 0\\ 0 & \varepsilon_0+\varepsilon_1 & 0\\ 0 & 0 & \varepsilon_0\end{bmatrix}\partial_t^2\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}=\begin{bmatrix}\partial_y^2+\partial_z^2 & -\partial_x\partial_y & -\partial_x\partial_z\\ -\partial_x\partial_y & \partial_z^2+\partial_x^2 & -\partial_y\partial_z\\ -\partial_x\partial_z & -\partial_y\partial_z & \partial_x^2+\partial_y^2\end{bmatrix}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}$$ 波动方程

$\vec{E}$ 是平面波

$$\vec{E}(\vec{r},t)=\begin{bmatrix}E_x(\vec{r},t)\\E_y(\vec{r},t)\\E_z(\vec{r},t)\end{bmatrix}=\begin{bmatrix}E_x^0\\E_y^0\\E_z^0\end{bmatrix}e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

求解 u

$$m\begin{bmatrix}\ddot{u}_x(\vec{r},t)\\\ddot{u}_y(\vec{r},t)\end{bmatrix}=-m\omega_0^2\begin{bmatrix}u_x(\vec{r},t)\\u_y(\vec{r},t)\end{bmatrix}+q\begin{bmatrix}E_x^0\\E_y^0\end{bmatrix}e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\Rightarrow\begin{bmatrix}u_x\\u_y\end{bmatrix}=\frac{q}{m(\omega_0^2-\omega^2)}\begin{bmatrix}E_x^0\\E_y^0\end{bmatrix}e^{i(\vec{k}\cdot\vec{r}-\omega t)}=\frac{q}{m(\omega_0^2-\omega^2)}\begin{bmatrix}E_x(\vec{r},t)\\E_y(\vec{r},t)\end{bmatrix}$$

电流密度 $\vec{J}=n_e q\dot{\vec{u}}$

$$\vec{J}=\begin{bmatrix}J_x(\vec{r},t)\\J_y(\vec{r},t)\\J_z(\vec{r},t)\end{bmatrix}=n_e q\begin{bmatrix}\dot{u}_x\\\dot{u}_y\\\dot{u}_z\end{bmatrix}=n_e q\begin{bmatrix}\alpha\dot{E}_x\\\alpha\dot{E}_y\\0\end{bmatrix}$$

$$=\begin{bmatrix}n_e q\alpha & 0 & 0\\0 & n_e q\alpha & 0\\0&0&0\end{bmatrix}\begin{bmatrix}\dot{E}_x\\\dot{E}_y\\\dot{E}_z\end{bmatrix}$$

M4

$\nabla\times\vec{B}=\mu(\vec{J}+\varepsilon\partial_t\vec{E})$

$$\vec{\nabla}\times\begin{bmatrix}B_x\\B_y\\B_z\end{bmatrix}=\mu_0\left(\varepsilon_0\partial_t\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}+\begin{bmatrix}J_x\\J_y\\J_z\end{bmatrix}\right)$$

$$=\mu_0\left(\varepsilon_0\begin{bmatrix}\dot{E}_x\\\dot{E}_y\\\dot{E}_z\end{bmatrix}+\begin{bmatrix}n_e q\alpha & 0 & 0\\0 & n_e q\alpha & 0\\0&0&0\end{bmatrix}\begin{bmatrix}\dot{E}_x\\\dot{E}_y\\\dot{E}_z\end{bmatrix}\right)$$

$$=\mu_0\begin{bmatrix}\varepsilon_0+\varepsilon_1 & 0 & 0\\ 0 & \varepsilon_0+\varepsilon_1 & 0\\ 0 & 0 & \varepsilon_0\end{bmatrix}\begin{bmatrix}\dot{E}_x\\\dot{E}_y\\\dot{E}_z\end{bmatrix}\quad(\varepsilon_1=n_e q\alpha)$$

M3 & M4

$$\begin{cases}\nabla\times\vec{E}=-\partial_t\vec{B}\\ \nabla\times\vec{B}=\mu_0\vec{\varepsilon}\partial_t\vec{E}\end{cases}$$

其中

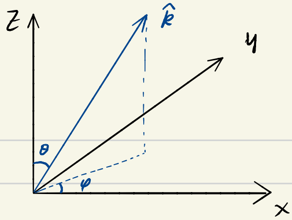
$$\vec{\varepsilon}=\begin{bmatrix}\varepsilon_0+\varepsilon_1 & 0 & 0\\ 0 & \varepsilon_0+\varepsilon_1 & 0\\ 0 & 0 & \varepsilon_0\end{bmatrix}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E})-\partial_t(\vec{\nabla}\times\vec{B})=-\partial_t(\vec{\nabla}\times\vec{B})-\mu_0\vec{\varepsilon}\partial_t^2\vec{E}^2\Rightarrow\vec{\nabla}\times\vec{\nabla}\times\vec{E}=-\mu_0\vec{\varepsilon}\partial_t^2\vec{E}$$

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E}=\vec{\nabla}(\vec{\nabla}\cdot\vec{E})-\nabla^2\vec{E}=\vec{\nabla}(\partial_xE_x+\partial_yE_y+\partial_zE_z)-(\partial_x^2+\partial_y^2+\partial_z^2)\vec{E}$$

$$=\begin{bmatrix}\partial_x^2E_x+\partial_x\partial_yE_y+\partial_x\partial_zE_z\\\partial_x\partial_yE_x+\partial_y^2E_y+\partial_y\partial_zE_z\\\partial_x\partial_zE_x+\partial_y\partial_zE_y+\partial_z^2E_z\end{bmatrix}-(\partial_x^2+\partial_y^2+\partial_z^2)\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}=\begin{bmatrix}-\partial_y^2-\partial_z^2 & \partial_x\partial_y & \partial_x\partial_z\\\partial_x\partial_y & -\partial_x^2-\partial_z^2 & \partial_y\partial_z\\\partial_x\partial_z & \partial_y\partial_z & -\partial_x^2-\partial_y^2\end{bmatrix}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}$$

$$\boxed{\mu_0\begin{bmatrix}\varepsilon_0+\varepsilon_1 & 0 & 0\\ 0 & \varepsilon_0+\varepsilon_1 & 0\\ 0 & 0 & \varepsilon_0\end{bmatrix}\partial_t^2\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}=\begin{bmatrix}\partial_y^2+\partial_z^2 & -\partial_x\partial_y & -\partial_x\partial_z\\ -\partial_x\partial_y & \partial_z^2+\partial_x^2 & -\partial_y\partial_z\\ -\partial_x\partial_z & -\partial_y\partial_z & \partial_x^2+\partial_y^2\end{bmatrix}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}}$$ 波动方程



$$\begin{bmatrix}k_x\\k_y\\k_z\end{bmatrix} = |k|\begin{bmatrix}\sin\theta\cos\varphi\\ \sin\theta\sin\varphi\\ \cos\theta\end{bmatrix}$$

求解波函数

预解式代入波动方程

① 验证预解式的函数形式满足方程（平面波解 $\vec{E} = \begin{bmatrix}E_x^0\\E_y^0\\E_z^0\end{bmatrix} e^{i(\vec{k}\cdot\vec{r}-\omega t)}$）

② 求出参数制约关系

- 参数 $\{(E_x^0, E_y^0, E_z^0), (k_x, k_y, k_z), \omega\} \Rightarrow (|E|, \hat{E}, |k|, \hat{k}, \omega)$
- 目标制约关系（谁依赖谁）
  - 自由参数 $(\omega, \hat{k}) \Rightarrow$ 考察给定频率给定方向的光
  - 被制约参数 $|E|, \hat{E}, |k|$

预解式

$$\vec{E} = \begin{bmatrix}E_x^0\\E_y^0\\E_z^0\end{bmatrix} e^{i(\vec{k}\cdot\vec{r}-\omega t)} = |E|\begin{bmatrix}\hat{E}_x\\\hat{E}_y\\\hat{E}_z\end{bmatrix} e^{i(|k|\hat{k}\cdot\vec{r}-\omega t)}$$

代入波动方程

$$LHS = \mu_0\overleftrightarrow{\varepsilon}\partial_t^2\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix} = (-\omega^2)\mu_0\overleftrightarrow{\varepsilon}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix} = -\mu_0\omega^2\begin{bmatrix}\varepsilon_0+\varepsilon_1 & & \\ & \varepsilon_0+\varepsilon_1 & \\ & & \varepsilon_0\end{bmatrix}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}$$

$$RHS = \begin{bmatrix}\partial_y^2+\partial_z^2 & -\partial_x\partial_y & -\partial_x\partial_z\\ -\partial_x\partial_y & \partial_z^2+\partial_x^2 & -\partial_y\partial_z\\ -\partial_x\partial_z & -\partial_y\partial_z & \partial_x^2+\partial_y^2\end{bmatrix}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix} = -\begin{bmatrix}k_y^2+k_z^2 & -k_xk_y & -k_xk_z\\ -k_xk_y & k_z^2+k_x^2 & -k_yk_z\\ -k_xk_z & -k_yk_z & k_x^2+k_y^2\end{bmatrix}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}$$

$LHS = RHS$

$$\mu_0\omega^2\begin{bmatrix}\varepsilon_0+\varepsilon_1 & & \\ & \varepsilon_0+\varepsilon_1 & \\ & & \varepsilon_0\end{bmatrix}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix} = |k|^2\begin{bmatrix}1-\sin^2\theta\cos^2\varphi & -\sin^2\theta\sin\varphi\cos\varphi & -\sin\theta\cos\theta\cos\varphi\\ -\sin^2\theta\sin\varphi\cos\varphi & 1-\sin^2\theta\sin^2\varphi & -\sin\theta\cos\theta\sin\varphi\\ -\sin\theta\cos\theta\cos\varphi & -\sin\theta\cos\theta\sin\varphi & \sin^2\theta\end{bmatrix}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}$$

齐次线性方程组的非平庸解

制约关系 $(|E|, \hat{E}, |k|)$ 依赖 $(\hat{k}(\theta,\varphi), \omega)$

齐次方程整理成

$$\overleftrightarrow{J}\begin{bmatrix}E_x^0\\E_y^0\\E_z^0\end{bmatrix} = \alpha\begin{bmatrix}E_x^0\\E_y^0\\E_z^0\end{bmatrix} \Rightarrow \text{求出依赖关系}$$

↓

方法：

$$\mu_0\omega^2\overleftrightarrow{\varepsilon}\begin{bmatrix}E_x^0\\E_y^0\\E_z^0\end{bmatrix} = |k|\overleftrightarrow{k}\begin{bmatrix}E_x^0\\E_y^0\\E_z^0\end{bmatrix} \Rightarrow \underbrace{\mu_0\omega^2(\overleftrightarrow{k})^{-1}\overleftrightarrow{\varepsilon}}_{\text{矩阵}}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix} = |k|\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}$$

$\Rightarrow |k|$ 是矩阵 $\mu_0\omega^2(\overleftrightarrow{k})^{-1}\overleftrightarrow{\varepsilon}$ 的本征值

$\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}$ 是矩阵 $\mu_0\omega^2(\overleftrightarrow{k})^{-1}\overleftrightarrow{\varepsilon}$ 的本征矢

$\Rightarrow$ 得到 $|k|$ 和 $\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}$ 关于 $(\omega, \theta, \varphi)$ 的依赖关系（见下页）

电场波函数

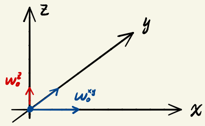
$$\vec{E} = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} e^{i(|k|\hat{k}\cdot\vec{r} - \omega t)} = |E| \begin{bmatrix} \hat{E}_x \\ \hat{E}_y \\ \hat{E}_z \end{bmatrix} e^{i(|k|\hat{k}\cdot\vec{r} - \omega t)}$$

$\mu_0 \omega^2 (\hat{\varepsilon})^{-1}$ 之的本征值&本征态：

$$|k_1| = \frac{\omega\sqrt{\varepsilon_0 + \varepsilon_1}}{c} \qquad \vec{E}_1 \propto \begin{bmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{bmatrix}$$

$$|k_2| = \frac{\omega}{c}\sqrt{\frac{2(\varepsilon_0 + \varepsilon_1)\varepsilon_0}{2\varepsilon_0 + \varepsilon_1 - \varepsilon_1\cos 2\theta}} \qquad \vec{E}_2 \propto \begin{bmatrix} \varepsilon_0\cos\theta\cos\varphi \\ \varepsilon_0\cos\theta\sin\varphi \\ -(\varepsilon_0+\varepsilon_1)\sin\theta \end{bmatrix}$$

物理诠释：在双折射介质中，给定 $(\omega, \hat{k})$ 的光
它的偏振方向只能取2个特定的方向 $\begin{bmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{bmatrix}$ 和 $\begin{bmatrix} \varepsilon_0\cos\theta\cos\varphi \\ \varepsilon_0\cos\theta\sin\varphi \\ -(\varepsilon_0+\varepsilon_1)\sin\theta \end{bmatrix}$
同时不同偏振方向对应的 $|k|$ 是不同的



$\Rightarrow$ 介电常数 $\overleftrightarrow{\varepsilon} = \begin{bmatrix} \varepsilon_{xy} & & \\ & \varepsilon_{xy} & \\ & & \varepsilon_z \end{bmatrix}$

双折射的性质和现象

性质

介质的性质：光学中，介质的性质由折射率唯一描述

双折射 $\Leftrightarrow$ 双折射率 $\Leftrightarrow$ 介质中存在两个折射率

回顾折射率的定义：$n = \frac{c}{v_p} = \frac{c}{\omega}|k| \propto |k|$

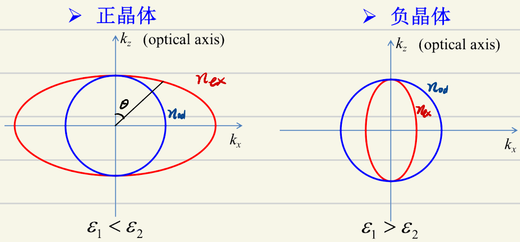
双折射晶体 { 正常光 / 反常光 }

$n_{od} = \frac{c}{\omega}|k_{od}| = c\sqrt{\mu_0 \varepsilon_{xy}}$

$n_{ex} = \frac{c}{\omega}|k_{ex}| = c\sqrt{\frac{2\mu_0\varepsilon_{xy}\varepsilon_z}{\varepsilon_{xy}+\varepsilon_z+(\varepsilon_z-\varepsilon_{xy})\cos 2\theta}} \xrightarrow[\text{光沿主轴传播}]{\theta=0} c\sqrt{\mu_0\varepsilon_{xy}}$

$\Rightarrow$ 光在介质中传播，可能感受到两种折射率，依赖于光的偏振

反射光的折射率依赖于波矢方向 (θ)



当光沿主轴(即z轴)传播时，反常光退化为正常光，即 $n_{ex}(\theta=0) = n_{od}(\theta=0)$

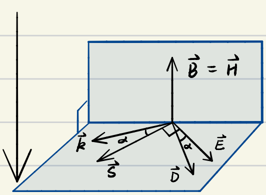
传播光的性质和现象

光的性质：传播方向、偏振方向、传播速度、光的强度

传播方向和偏振方向

传播方向 { 波矢方向 $\vec{k}$ / 能流方向 $\vec{S} \propto \vec{E}\times\vec{H}$ (能流方向是真正的传播方向) }

偏振方向 $\vec{E} \oplus \vec{D}$，$\vec{H}$，$\vec{B}$



电场方向 $\leftrightarrow$ 传播方向 $\leftrightarrow$ 磁场方向；电场方向 $\updownarrow$ 磁场方向

$\vec{E}.\vec{D} \leftrightarrow \vec{k}.\vec{S} \leftrightarrow \vec{B}.\vec{H}$；$\vec{E}.\vec{D} \updownarrow \vec{B}.\vec{H}$

推导 出发点：Maxwell's Equations & 平面波解

① $\vec{E}$、$\vec{D}$　由 $\vec{D} = \overleftrightarrow{\varepsilon}\vec{E} \Rightarrow \vec{D}$ 与 $\vec{E}$ 不平行，夹角为 α　$|\vec{D}\cdot\vec{E}| = |E||D|\cos\alpha \Rightarrow \cos\alpha = \frac{|\vec{D}\cdot\vec{E}|}{|D||E|}$

② $\vec{B}$、$\vec{H}$　$\vec{B} = \mu\vec{H}$，在一般介质中 $\mu = \mu_0 = 1 \Rightarrow$ $\vec{B} = \vec{H}$

③ 4个Maxwell方程

M1：$\vec{\nabla}\cdot\vec{D} = \rho = 0 \Rightarrow (i\vec{k})\cdot\vec{D} = 0 \Rightarrow \vec{k}\cdot\vec{D} = 0$　　$\vec{k}\perp\vec{D}$

M2：$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \vec{k}\cdot\vec{B} = 0$　　$\vec{k}\perp\vec{D}$

M3：$\vec{\nabla}\times\vec{E} = -\partial_t\vec{B} \Rightarrow (i\vec{k})\times\vec{E} = i\omega\vec{B} \Rightarrow \vec{k}\times\vec{E} = \omega\vec{B}$　　$\vec{B}\perp\vec{k}$　$\vec{B}\perp\vec{E}$

M4：$\vec{\nabla}\times\vec{H} = \partial_t\vec{D} \Rightarrow (i\vec{k})\times\vec{H} = -i\omega\vec{D} \Rightarrow \vec{k}\times\vec{H} = -\omega\vec{D}$　　$\vec{D}\perp\vec{k}$　$\vec{D}\perp\vec{H}$

④ 能流　$\vec{S} = \vec{E}\times\vec{H} \Rightarrow$　$\vec{S}\perp\vec{E}$　$\vec{S}\perp\vec{H}$

传播速度

群速度 v.s. 相速度

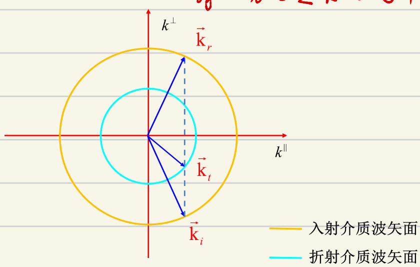
定义

相速度（相位的速度） $\vec{v}_p = \frac{\omega}{|k|}\cdot\hat{k}$

群速度 ⇒ 能流方向 $\vec{v}_g = \nabla_{\vec{k}}\,\omega(\vec{k}) = (\partial_{k_x}\omega(\vec{k}), \partial_{k_y}\omega(\vec{k}), \partial_{k_z}\omega(\vec{k}))$

$\vec{v}_g$ 的方向是 $\vec{k}$ 空间中等频面的法线方向

基于波矢中的等频面进行讨论



波矢匹配 $k_i^{\parallel} = k_r^{\parallel} = k_t^{\parallel} = k^{\parallel}$

波矢匹配判断波的方向

- 相速度方向：$\hat{k}$
- 群速度方向（能流方向）：相应 k 点的等频率面法向

△ 波矢匹配适用于所有波动

正常光和反常光的等频面

- 正常光
- 反常光

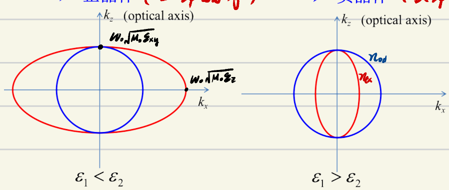
$\omega(k_x, k_y, k_z) = \omega_0$

$|k_{od}| = \omega_0\sqrt{\mu_0\varepsilon_{xy}} \Rightarrow k_x^2 + k_y^2 + k_z^2 = \omega_0^2\mu_0\varepsilon_{xy}$ 等频面方程

$|k_{ex}| = \omega_0\sqrt{\frac{2\varepsilon_{xy}\varepsilon_z}{\varepsilon_{xy}+\varepsilon_z+(\varepsilon_z-\varepsilon_{xy})\cos 2\theta}} \Rightarrow k_x^2 + k_y^2 + k_z^2 = \frac{2\omega_0^2\varepsilon_{xy}\varepsilon_z}{\varepsilon_{xy}+\varepsilon_z+(\varepsilon_z-\varepsilon_{xy})\cos 2\theta}$

θ 是 $\hat{k}$ 与 z 轴的夹角 $k_z = |k|\cos\theta \Rightarrow \cos 2\theta = 2\cos^2\theta - 1 = \frac{2k_z^2}{|k|^2} - 1$

➤ 正晶体（正常晶体） ➤ 负晶体（反常晶体）

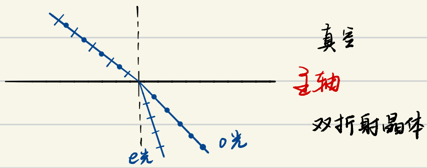


正常晶体 $\varepsilon_{xy} < \varepsilon_z \Rightarrow |k|_{od} \le |k|_{ex} \Rightarrow |v_p|_{od} \ge |v_p|_{ex}$

正常晶体中正常光跑得快

现象 光在双折射晶体表面的折射

系统



目标 求出双折射晶体中正常光和反常光的折射角

出发点 波矢空间等频面的波矢匹配条件

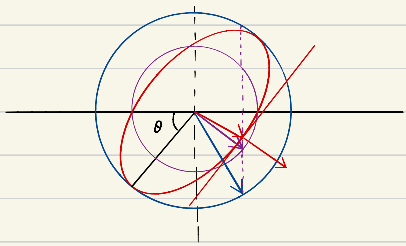
步骤

① 定义坐标系：沿袭入射问题中的定义

② 分别画出入射光和正常折射光、反常折射光的等频面

③ 根据波矢匹配条件确定正常折射光和反常折射光的波矢方向

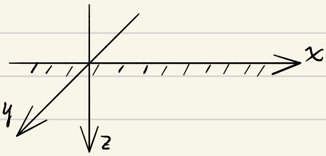
④ 根据等频面进一步确定反常光的能流方向（真折射方向）



双折射晶体介面处的折射

折射的物理图像

系统



界面处的折射和反射

入射光激发界面上的Q做受激振荡 ⇒ Q辐射次级波

⇒ 次级波相长/相消干涉

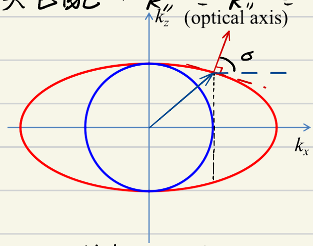
$$\begin{cases} \ddot{x} = k_x x + E_x^{in} \\ \ddot{y} = k_y y + E_y^{in} \end{cases}$$

(双折射晶体表面的)折射的定量描述

原则

波矢匹配：$k_\parallel^{in} = k_\parallel^{od} = k_\parallel^{ex}$

反常光的等频率面



$$\sigma = \arctan\frac{\varepsilon_{xy} k_0^x}{\varepsilon_z k_0^z}$$

$$|k|_{ex} = w_0\sqrt{\frac{2\mu_0\varepsilon_{xy}\varepsilon_z}{\varepsilon_{xy}+\varepsilon_z+(\varepsilon_z-\varepsilon_{xy})\cos 2\theta}}$$

$$k_z = |k|\cos\theta \Rightarrow \cos 2\theta = 2\cos^2\theta - 1 = \frac{2k_z^2}{|k|^2} - 1$$

$$\Rightarrow |k|^2\left[\varepsilon_{xy}+\varepsilon_z+(\varepsilon_z-\varepsilon_{xy})\left(\frac{2k_z^2}{|k|^2}-1\right)\right] = 2\mu_0 w_0^2\varepsilon_{xy}\varepsilon_z$$

$$\Rightarrow (\varepsilon_{xy}+\varepsilon_z)|k|^2 + (\varepsilon_z-\varepsilon_{xy})(2k_z^2-|k|^2) = 2\mu_0 w_0^2\varepsilon_{xy}\varepsilon_z$$

$$\Rightarrow \varepsilon_{xy}(k_x^2+k_y^2) + \varepsilon_z k_z^2 = \mu_0 w_0^2\varepsilon_{xy}\varepsilon_z$$

$$\Rightarrow \frac{k_x^2+k_y^2}{\mu_0 w_0^2\varepsilon_z} + \frac{k_z^2}{\mu_0 w_0^2\varepsilon_{xy}} = 1$$

hint：

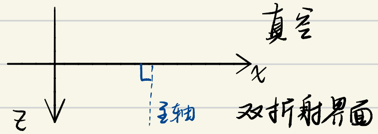
① $|k|^2 = k_x^2+k_y^2+k_z^2$

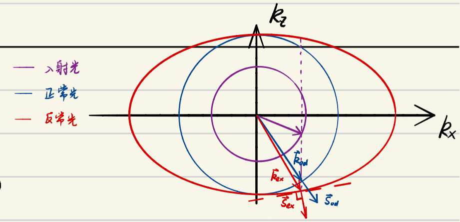
② $|k|\cos\theta = k_z$

折射示例

主轴相对界面

① 垂直



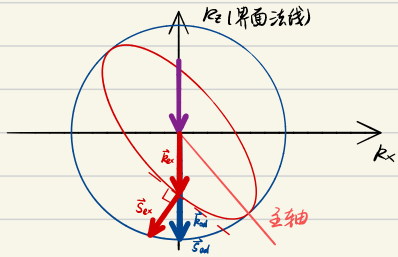


特殊情形：当入射光垂直界面入射时，正常光与反常光

{ 波矢重合（k_z轴上o光和e光等频面相交）

能流方向重合（k_z轴上o光和e光等频面相切）

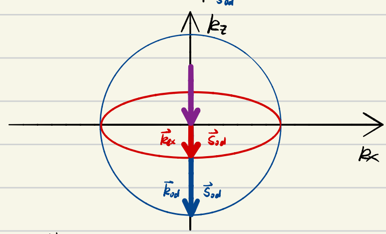
② 倾斜



即使垂直入射

o光和e光仍会分离

③ 平行



垂直入射时

o光与e光波矢和能流方向都平行

但 $|k_{od}| \neq |k_{ex}|$

数学描述（垂直入射）

$$\vec{E}_{od}(\vec{r},t) = |E_1|\begin{bmatrix}\sin\varphi\\ -\cos\varphi\\ 0\end{bmatrix} e^{i(|k_{od}|z-\omega t)} \xrightarrow[\varphi=0]{\theta=0} |E_1|\begin{bmatrix}0\\ -1\\ 0\end{bmatrix} e^{i\omega(\sqrt{\mu_0\varepsilon_1}\,z-t)} \sim TE$$

$$\vec{E}_{ex}(\vec{r},t) = |E_1|\begin{bmatrix}\varepsilon_2\cos\theta\cos\varphi\\ \varepsilon_2\cos\theta\sin\varphi\\ -\varepsilon_1\sin\theta\end{bmatrix} e^{i(|k_{ex}|z-\omega t)} \xrightarrow[\varphi=0]{\theta=0} \varepsilon_2|E_1|\begin{bmatrix}1\\ 0\\ 0\end{bmatrix} e^{i\omega(\sqrt{\mu_0\varepsilon_2}\,z-t)} \sim TM$$

{ 正常光和反常光传播方向平行

正常光和反常光偏振方向垂直 ⇒ 用来做波片

正常光和反常光 $|k|$不同 ⇒ n不同

双折射光学元器件

波片

回顾

波片的琼斯矩阵

入射光 $\begin{bmatrix} E_i^x \\ E_i^y \end{bmatrix}$　　出射光 $\begin{bmatrix} E_o^x \\ E_o^y \end{bmatrix} = \begin{bmatrix} e^{i\varphi_x} & 0 \\ 0 & e^{i\varphi_y} \end{bmatrix}\begin{bmatrix} E_i^x \\ E_i^y \end{bmatrix}$

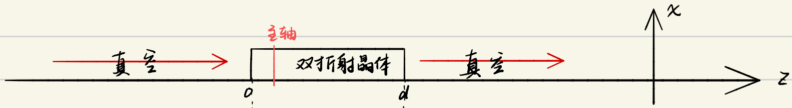
$e^{i\varphi_0}\begin{bmatrix} 1 & 0 \\ 0 & e^{i\Delta\varphi} \end{bmatrix}$

双折射

设置

光轴平行于界面；入射光垂直入射时，有延迟作用

系统



TE	$\begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{i(kz-\omega t)}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{i[k_{od}(z-0)-\omega t+\varphi_0]}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{i[k(z-d)-\omega t+\varphi_d^{od}]}$
TM	$\begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{i(kz-\omega t)}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{i[k_{ex}(z-0)-\omega t+\varphi_0]}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{i[k(z-d)-\omega t+\varphi_d^{ex}]}$

$\varphi_0 = 0$

$\varphi_d^{od} = k_{od}\cdot d + \varphi_0 = k_{od}\cdot d$　　$\varphi_d^{ex} = k_{ex}\cdot d + \varphi_0 = k_{ex}\cdot d$

$$\Rightarrow \quad E_i = \alpha E_{TE} + \beta E_{TM} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} e^{i(kz-\omega t)}$$

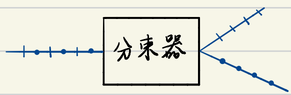
双折射晶体的J-矩阵

$$E_o = \alpha E_{TE} e^{i\varphi_d^{od}} + \beta E_{TM} e^{i\varphi_d^{ex}} = \begin{bmatrix} \alpha e^{i\varphi_d^{od}} \\ \beta e^{i\varphi_d^{ex}} \end{bmatrix} e^{i(kz-\omega t)} = \begin{bmatrix} e^{i\varphi_d^{od}} & 0 \\ 0 & e^{i\varphi_d^{ex}} \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} e^{i(kz-\omega t)}$$

对垂直入射光，厚度为d的双折射晶体（光轴平行界面）的作用可由J矩阵 $e^{i\varphi_d^{od}}\begin{bmatrix} 1 & 0 \\ 0 & e^{i\Delta\varphi} \end{bmatrix}$ 描述，即这块晶体起到了波片的作用

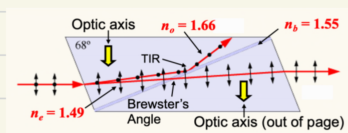
分束器

功能要求

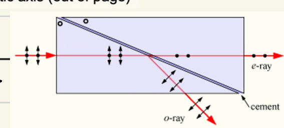


举例

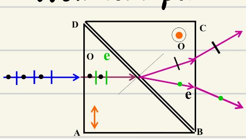
Nicol Prism →



Glan-Thompson and Glan-Air Polarizer

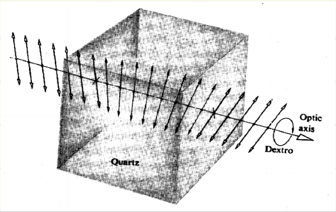


Wollaston polarizing beam splitter



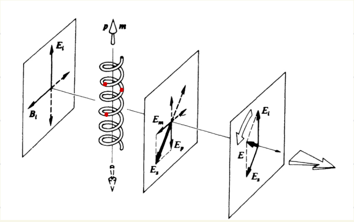
旋光性

现象描述



$$v \sim \begin{bmatrix} \cos kz \\ \sin kz \end{bmatrix}$$

机制



电荷束缚在螺旋线中运动

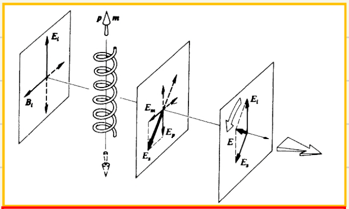
$\xrightarrow{\text{光入射}}$ $\begin{cases} \text{电荷沿 y 轴上下振荡(偶极辐射)} \Rightarrow E_p \\ \text{在 x-y 平面做圆周运动(同步辐射)} \Rightarrow E_m \end{cases}$

E_i 入射光偏振

E_s 螺线管次级光的偏振

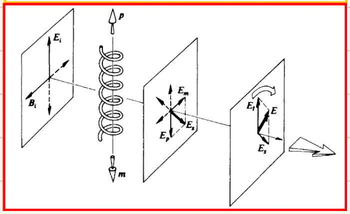
$\begin{cases} E_p \text{ 偶极辐射} \\ E_m \text{ 同步辐射} \end{cases}$

$\vec{E}_{out} = \vec{E}_i + \vec{E}_s$　　$\vec{E}_s = \vec{E}_m + \vec{E}_p$



left-handed material

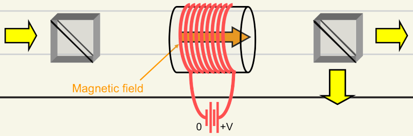
D-rotation



right-handed material

L-rotation

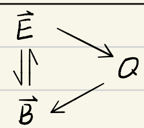
磁致旋光效应



总结

光学的2个范式

1



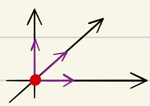
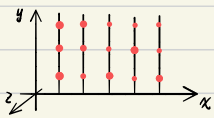
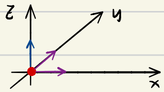
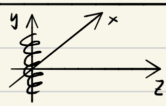
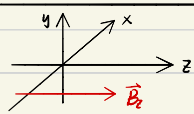
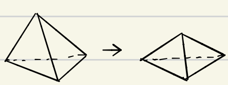
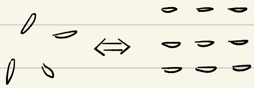
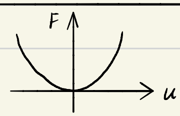
2 介质统一由折射率 n 描述

范式的意义

指导坚壁清野

指导打破范式

范式的集结

	Q	u	
色散		$\partial_t^2\begin{bmatrix}u_x\\u_y\\u_z\end{bmatrix}=-\omega_0^2\begin{bmatrix}u_x\\u_y\\u_z\end{bmatrix}+\frac{q}{m}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}$	$n(\omega)$
二向色性		$\partial_t^2\begin{bmatrix}u_x\\u_y\\u_z\end{bmatrix}=\begin{bmatrix}\infty&&\\&0&\\&&\infty\end{bmatrix}\begin{bmatrix}u_x\\u_y\\u_z\end{bmatrix}-\begin{bmatrix}0&&\\&\infty&\\&&0\end{bmatrix}\begin{bmatrix}\dot u_x\\\dot u_y\\\dot u_z\end{bmatrix}+\frac{q}{m}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}$	$n(\hat{E})$
双折射		$\partial_t^2\begin{bmatrix}u_x\\u_y\\u_z\end{bmatrix}=\begin{bmatrix}\omega_o^2&&\\&\omega_o^2&\\&&\omega_e^2\end{bmatrix}\begin{bmatrix}u_x\\u_y\\u_z\end{bmatrix}+\frac{q}{m}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}$	$n(\omega,\hat{k})$
旋光性			$n(\cdots,L_{\text{旋}})$ or $n(\cdots,R_{\text{旋}})$
F-效应（右旋致旋光）		$\begin{cases}\ddot u_x=-kx+E_x^{cir}(t)+q\dot u_y B_z\\ \ddot u_y=-ky+E_y^{cir}(t)-q\dot u_x B_z\end{cases}$	$n(\cdots,B)$
光弹性		$\begin{cases}\ddot u_x=-\omega_x^2(k)u_x+\cdots\\ \ddot u_y=-\omega_y^2(k)u_y+\cdots\end{cases}$	$n(\cdots,k)$
Kerr效应			$n(\cdots,\lvert E\rvert^2)$
P-效应	 非线性谐振子	$m\ddot u=k(u-u_0)^2=\underbrace{ku^2}_{\text{高阶小量}}-2ku_0u+\underbrace{ku_0^2}_{\text{常数}}$ $u\ll1$ 微振动 $\Rightarrow m\ddot u\approx-2ku_0u$	$n(\cdots,\lvert E\rvert)$